FUNZIONI
MONOTONIA
LIMITI
DERIVATE
SUCCESSIONI

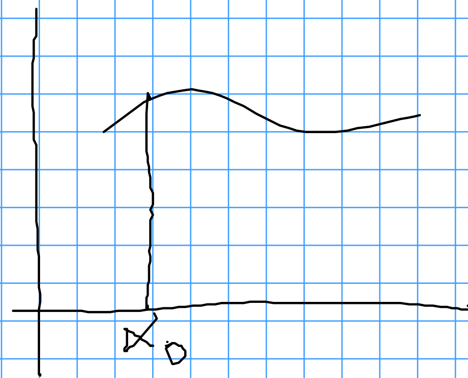
$a_n : \mathbb{N} \longrightarrow \mathbb{R}$

$a_n = n^2 + n$

$$\begin{cases} a_0 = 0 \\ a_n = a_{n-1} + n \end{cases}$$

$$\frac{a_n - a_{n-1}}{1} = n$$

$$f : \mathbb{R} \to \mathbb{R}$$



$$\frac{f(x_0+h) - f(x_0)}{h}$$

$$f(x) = ax^2 + bx + c$$

$$\frac{a(x+h)^2 + b(x+h) + c - (ax^2 + bx + c)}{h} =$$

$$\left(\cancel{ax^2} + 2axh + ah^2 + \cancel{bx} + bh \cancel{+c} - \cancel{ax^2} - \cancel{bx} - \cancel{c}\right)\frac{1}{h} =$$

$$\frac{2ax\cancel{h} + ah^{\cancel{2}} + b\cancel{h}}{\cancel{h}} = 2ax + b$$

$$a_n = \alpha n^2 + \beta n + \cancel{\gamma} \leftarrow \text{TEST}$$

$$\alpha n^2 + \beta n = \alpha(n-1)^2 + \beta(n-1) + n$$

$$\cancel{\alpha n^2} + \cancel{\beta n} = \cancel{\alpha n^2} - \underline{2\alpha n} + \alpha + \cancel{\beta n} - \beta + \underline{n}$$

$$n(1 - 2\alpha) + (\alpha - \beta) = 0n + 0$$

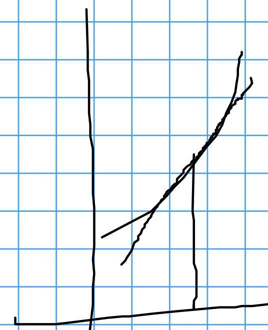
$$1 - 2\alpha = 0 \Longrightarrow \alpha = \frac{1}{2} \qquad \beta = \frac{1}{2}$$

$$a_n = \frac{n^2}{2} + \frac{n}{2} = \frac{n(n+1)}{2}$$

$$\begin{cases} a_0 = 0 \\ a_n = a_{n-1} + n^2 \end{cases}$$

$$\alpha n^3 + \beta n^2 + \gamma n$$

$$\begin{cases} a_0 = 1 \\ a_1 = 1 \\ a_n = a_{n-1} + a_{n-2} \end{cases}$$



$$a_n - a_{n-1} = a_{n-2}$$

$$\frac{e^{x+h} - e^x}{h} = e^x \left( \frac{e^h - 1}{h} \right) \to e$$

$e$ numero di Neper

$$a_n = a_{n-1} + a_{n-2}$$

$\boxed{\text{Test} \quad b^n}$ $\quad b^n - b^{n-1} - b^{n-2} = 0$

$$b^{n-2}\left(b^2 - b - 1\right) = 0$$

$$b_{12} = \frac{1 \mp \sqrt{5}}{2}$$

$$a_n = a_{n-1} + a_{n-2}$$

$x, y$

$$x(n) = x(n-1) + x(n-2)$$
$$y(n) = y(n-1) + y(n-2)$$

$$\alpha \left( \frac{1-\sqrt{5}}{2} \right)^n + \beta \left( \frac{1+\sqrt{5}}{2} \right)^n = a_n$$

$$\begin{cases} \alpha + \beta = 1 & (a_0 = 1) \\ \alpha \left( \frac{1-\sqrt{5}}{2} \right) + \beta \left( \frac{1+\sqrt{5}}{2} \right) = 1 & (a_1 = 1) \end{cases}$$

$\mathbb{N}$ $\quad 0 \in \mathbb{N}$

$1 \in \mathbb{N}$ PEANO

$\Sigma \quad \sum_{i=0}^{n} i = 0+1+2+\_\_+n$

$= \frac{n(n+1)}{2}$

$S(0) = 0$ vero

è vero $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$

$\Rightarrow$ devo dim è vero $\sum_{i=0}^{n+1} i$

$$\sum_{i=0}^{n+1} i = \sum_{i=0}^{n} i + n+1 = \frac{n(n+1)}{2} + n+1 =$$

$$= (n+1)\left(\frac{n}{2}+1\right) = \frac{(n+1)(n+2)}{2}$$

$$\boxed{\sum_{i=1}^{n} i^2 = \frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}}$$ ip. induttiva

$$\frac{(n+1)^3}{3}+\frac{(n+1)^2}{2}+\frac{(n+1)}{6}$$

è vero $n = 1$ ✓

$$\sum_{i=1}^{n+1} i^2 = \sum_{i=1}^{n} i^2 + (n+1)^2 = \frac{n^3}{3}+\frac{n^2}{2}+\frac{n}{6}+n^2+2n+1$$

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$$\sum_{k=1}^{n} \left(3k(k-1)+1\right) = n^3$$

$n = 1 \quad 1 = 1$ vero

$$\sum_{k=1}^{n+1}(3k(k-1)+1)=\sum_{k=1}^{n}(3k(k-1)+1)+$$

$$+3(n+1)(n+1-1)+1=n^3+3n^2+3n+1=$$

$$=(n+1)^3$$

$$\sum_{k=1}^{n}k\cdot k!=(n+1)!-1$$

$$\sum_{k=1}^{n+1}k\cdot k!=(n+1)!-1+(n+1)(n+1)!$$

$$=(n+1)!(n+2)-1$$

$$(n+2)!-1$$