NUMERI COMPLESSI

QUADRILATERI CICLICI

RADICI N-ESIME DELL'UNITÀ

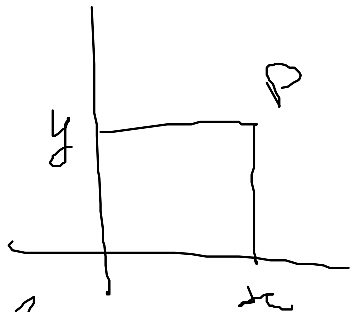
TEOREMA DI NAPOLEONE

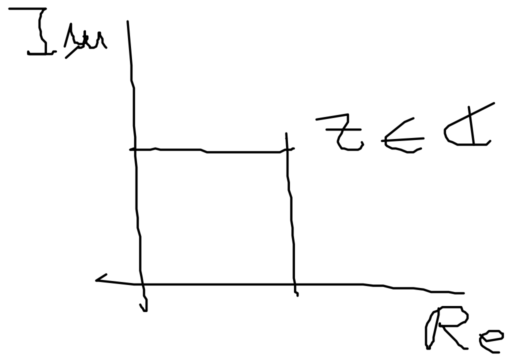
$$x^2+1=0$$
$$i^2=-1$$

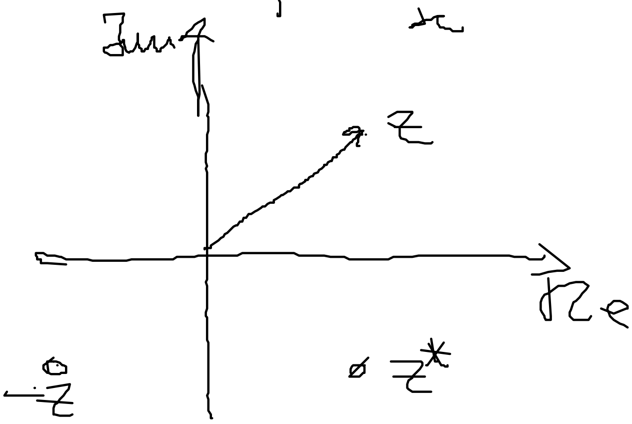
$$i=i$$
$$i^2=-1$$
$$i^3=-i$$
$$i^4=1$$
$$i^5=i$$

$$x+iy$$

$\mathbb{C}$



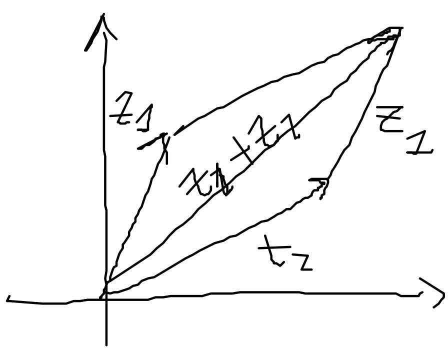




$$z=x+iy$$
$$z^*=x-iy$$

$$(z_1+z_2)^*=z_1^*+z_2^*$$

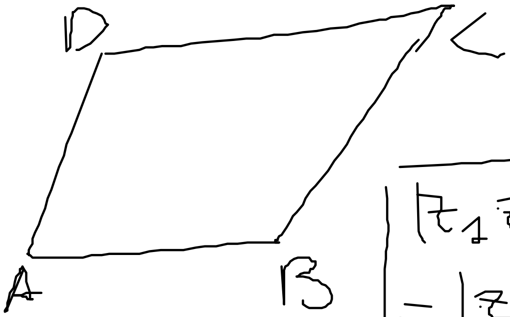
$$|\cdot| \quad \mathbb{C}\longrightarrow \mathbb{R}^+$$

$$|z|=\sqrt{x^2+y^2} \qquad |z|^2=z z^*=(x+iy)(x-iy)$$
$$=x^2-(i^2)y^2=x^2+y^2$$




$$|z_1|+|z_2|\geq|z_1+z_2|$$

Теорема Птолемея




$A \quad B \quad C \quad D$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$a \quad b \quad c \quad d$

$$|AB||CD| + |BC||AD|$$

$$\geq |AC||BD|$$

$$\boxed{|z_1 z_2| = |z_1||z_2|}$$

$$|a-b||c-d| + |b-c||a-d| \geq |a-c||b-d|$$

$$|\underbrace{(a-b)(c-d)}_{z_1}| + |\underbrace{(b-c)(a-d)}_{z_2}| \geq$$

$$\geq |z_1 + z_2|$$

$$(a-b)(c-d) + (b-c)(a-d) =$$

$$\cancel{ac} - bc - \underbrace{da} + \cancel{bd} + \underbrace{ba} - \cancel{ac} - \cancel{bd} + cd =$$

$$a(b-d) + c(d-b) = a(b-d) - c(b-d)$$

$$|(b-d)(a-c)|$$

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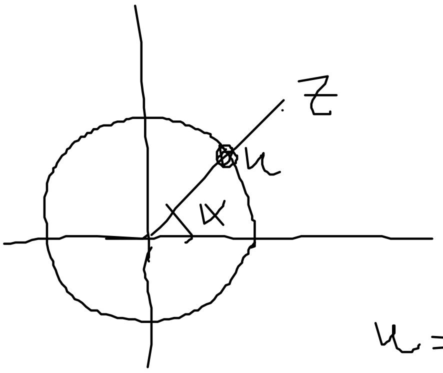



$$z = x + iy \qquad |z|$$

$$z = |z|\underbrace{\left(\frac{x}{|z|} + i\frac{y}{|z|}\right)}_{u \in U}$$

$$u = \cos\alpha + i\sin\alpha$$

$$z = |z|(\cos\alpha + i\sin\alpha) \qquad \alpha = \arg(z)$$

$$u_1, u_2 \qquad u_1 u_2 =$$

$$(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta) =$$

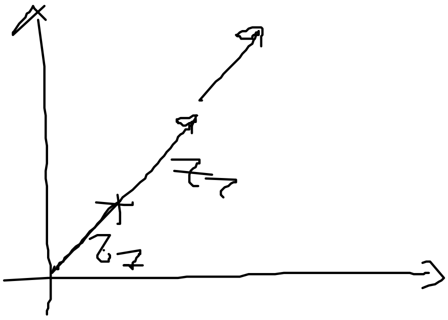
$$\cos\alpha\cos\beta - \sin\alpha\sin\beta + i\sin\alpha\cos\beta + i\sin\beta\cos\alpha$$

$$(\cos\alpha\cos\beta - \sin\alpha\sin\beta) + i(\sin\alpha\cos\beta + \sin\beta\cos\alpha)$$

$$\cos(\alpha+\beta) + i\sin(\alpha+\beta)$$

$$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$$

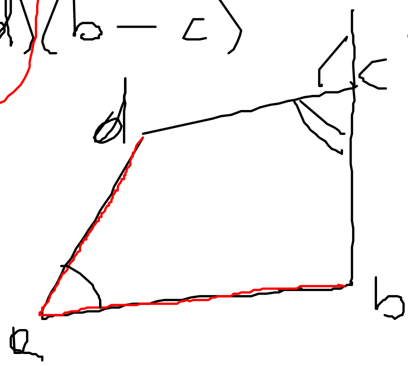
$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$




$$z_1 = k z_2 \qquad k \in \mathbb{R}^+$$

$$(a-b)(c-d) + (b-c)(a-d)$$

$$\frac{(a-b)(c-d)}{(a-d)(b-c)} = \underbrace{\frac{a-b}{a-d}}_{z_1}\left(\underbrace{-\frac{c-d}{c-b}}_{z_2}\right)$$




$z^n = 1$ $\qquad$ $z = 1$

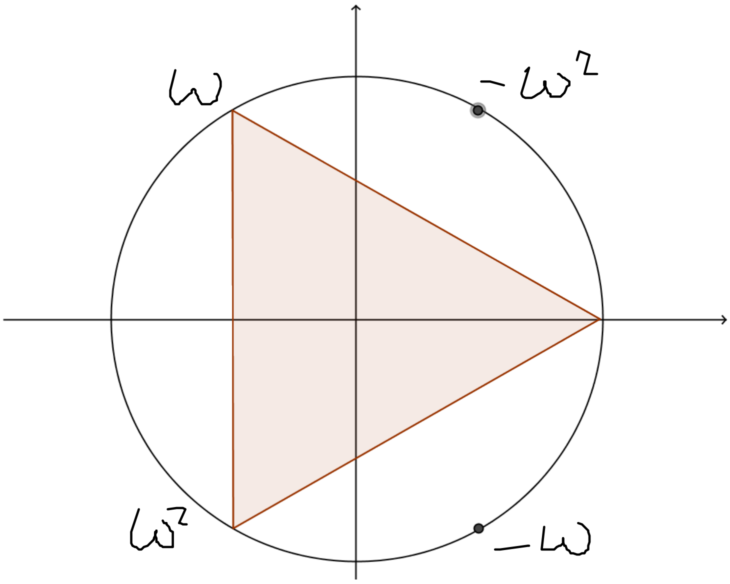
$z^2 = 1$ $\qquad$ $z_0 = 1 \quad z_1 = -1$

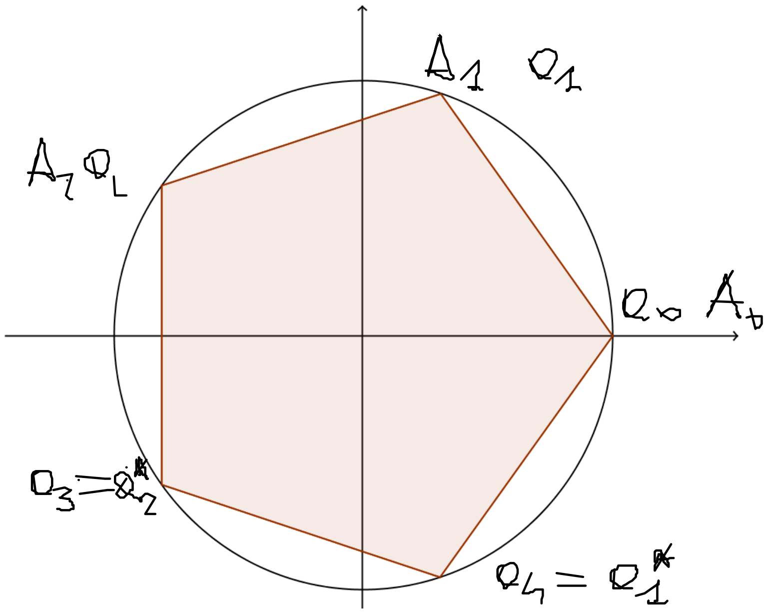
$z^3 = 1$ $\qquad$ $1 = \cos 2k\pi + i\sin 2k\pi$

$$z = \frac{1}{2} + i\frac{\sqrt{3}}{2} \qquad \arg(z) = \frac{\pi}{3} \qquad \arg(z^2) = 2\frac{\pi}{3}$$

$$z_k = \cos\frac{2k\pi}{3} + i\sin\frac{2k\pi}{3} \qquad k = 0, 1, 2$$

$$z_0 = 1 \qquad z_1 = -\frac{1}{2} + i\frac{\sqrt{3}}{2} \qquad z_2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$




$$\overline{A_0A_1}^2\,\overline{A_0A_2}^2 = 5$$

$$|a_1-a_0|^2\,|a_2-a_0|^2$$

$$(a_1-a_0)(a_1-a_0)^*$$

$$(a_2-a_0)(a_2-a_0)^*$$

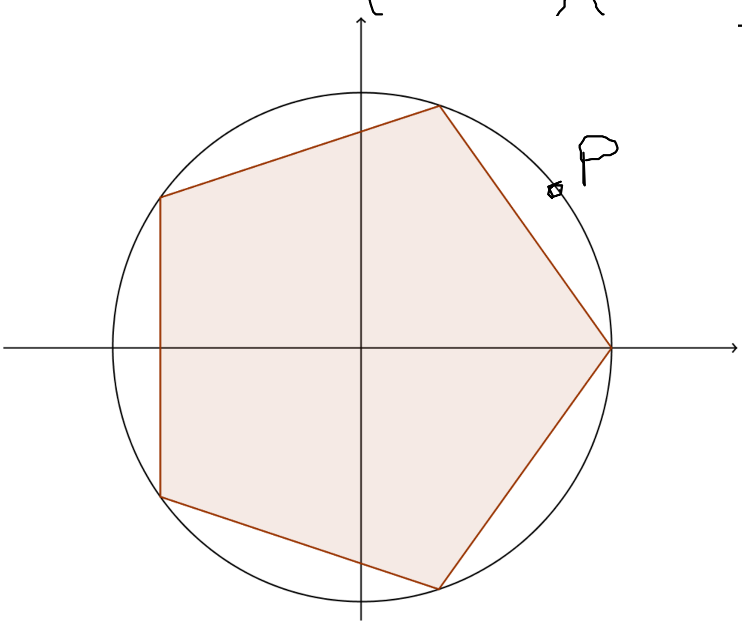
$$(a_1-a_0)(a_1^*-a_0)(a_2-a_0)(a_2^*-a_0)$$

$$(a_1-a_0)(a_4-a_0)(a_2-a_0)(a_3-a_0)$$

$$\omega^5-1=(\omega-1)(\omega^4+\omega^3+\omega^2+\omega+1)$$

$A_0, A_1, A_2, \dots A_{n-1}$

$A_k$




$$\sum_k \overline{PA_k}^2 =$$

$$\overline{PA_k}^2 = |z - a_k|^2$$

$$|z-a_k|^2 = (z-a_k)(z^* - a_k^*) =$$

$$= \underset{\overset{\shortparallel}{1}}{zz^*} - z^* a_k - z a_k^* + \underset{\overset{\shortparallel}{1}}{a_k a_k^*}$$

$$\sum_k (2 - z^* a_k - z a_k^*) = 2n$$

$$x^2 - sx + p$$

$$(x-\alpha)(x-\beta)(x-\gamma)$$

$$x^3 - x^2(\alpha+\beta+\gamma) + \cdot$$

$$(z^n - a^n) = z^n - 1$$

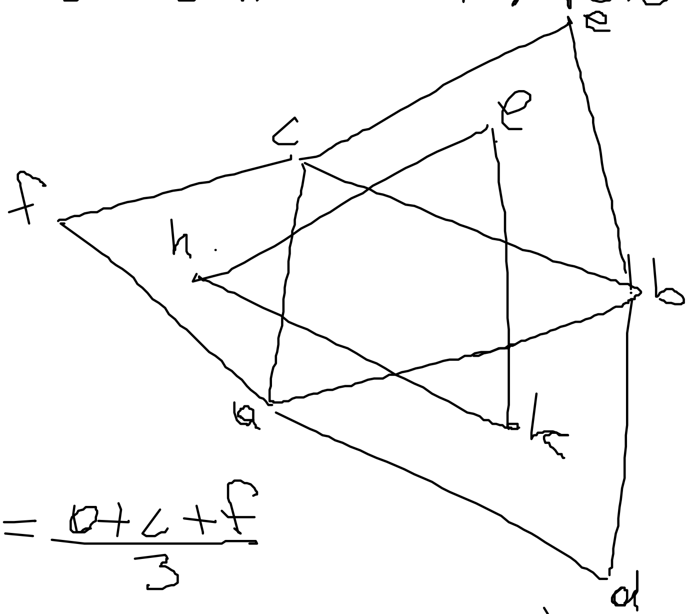
$$(z-a)(\quad\quad)$$

$$(z-a_0)(z-a_1) \cdot \quad (z-a_{n-1})$$

$$z^n + z^{n-1}\left(\underbrace{\sum a_k}_{0}\right) + \cdot \quad \cdot$$

TEOREMA di NAPOLEONE

JULIA




$$d = (b-a)(-\omega)$$

$$e = (c-b)(-\omega)$$

$$f = (c-a)(-\omega^2)$$

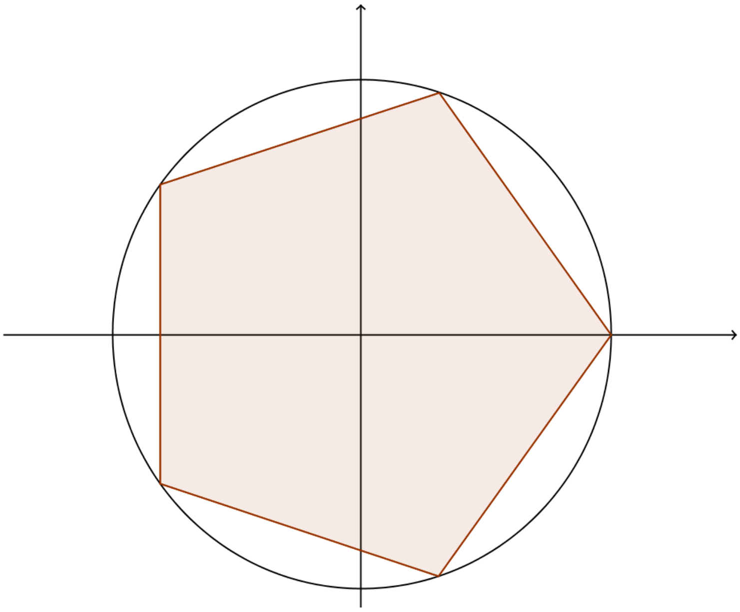
$$h = \frac{a+c+f}{3}$$

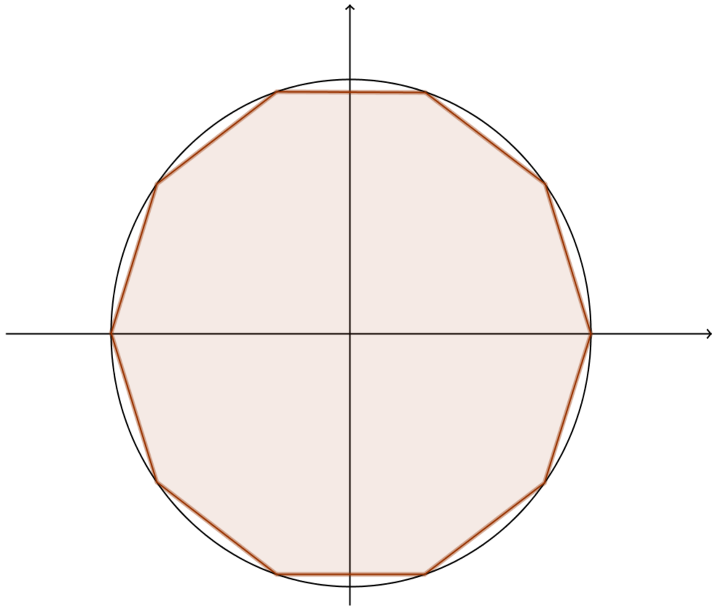
$$h = \frac{a+c+(c-a)(-\omega^2)}{3} \qquad e = \frac{c+b+(c-b)(-\omega)}{3}$$

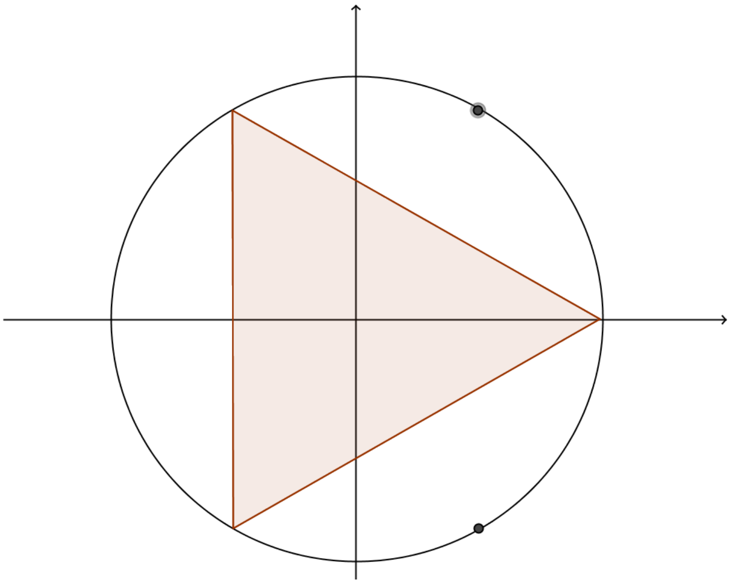
$$k = \frac{a+b+(b-a)(-\omega)}{3}$$

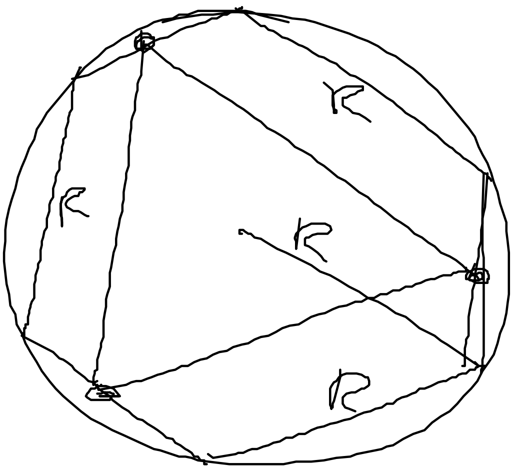
$$\omega = e^{+i\frac{\pi}{3}}$$

$$\omega = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$$





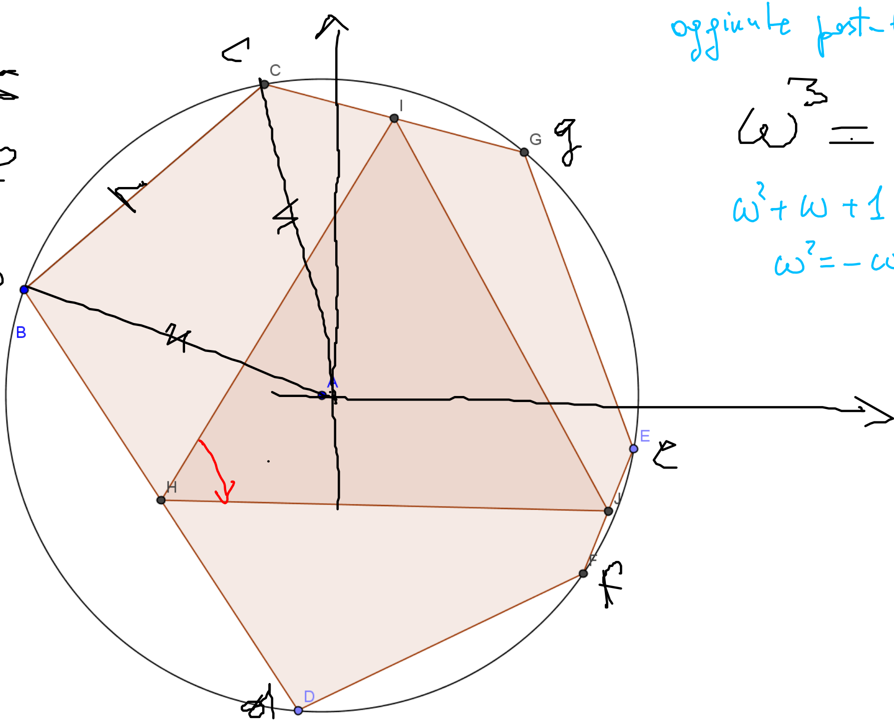



r
r
r
r

$b(-\omega) = c$

$g(-\omega) = e$

$f(-\omega) = d$




aggiunte post-lezione

$\omega^3 = 1$

$\omega^2 + \omega + 1 = 0$

$\omega^2 = -\omega - 1$

$c = -\omega b \qquad i = \dfrac{c+g}{2} = \dfrac{g-\omega b}{2}$

$e = -\omega g \qquad h = \dfrac{d+b}{2} = \dfrac{b-\omega f}{2}$

$d = -\omega f \qquad j = \dfrac{f+e}{2} = \dfrac{f-\omega g}{2}$

vogliamo dimostrare che $(i-h)(-\omega) = j-h$

$$j-h = \frac{1}{2}(f-\omega g - b + \omega f) = \boxed{\frac{1}{2}\left(f-b-\omega(g-f)\right)}$$

$$(i-h)(-\omega) = \frac{1}{2}(g-\omega b - b + \omega f)(-\omega) =$$

$$= \frac{1}{2}\left(-\omega(g-b) + \omega^2(b-f)\right) = \frac{1}{2}\left(-\omega g + \cancel{\omega b} - b + f - \cancel{\omega b} + \omega f\right) =$$

$$= \boxed{\frac{1}{2}\left(f-b-\omega(g-f)\right)}$$