Giovedì 5 novembre

GEOMETRIA

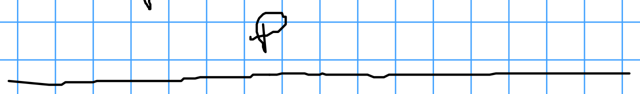
geometria euclidea (V postulato)

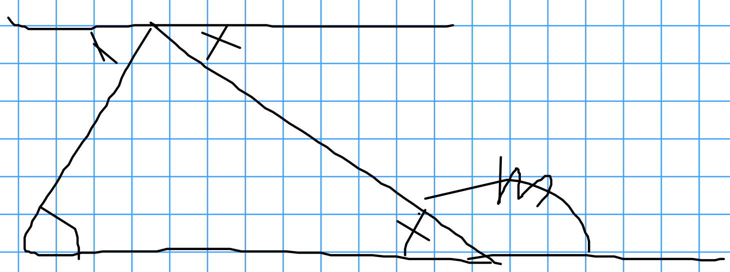
Teorema di TALETE

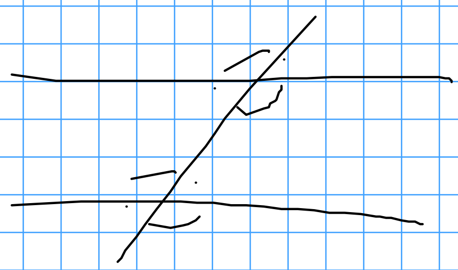
Similitudini (OMOTETIE)

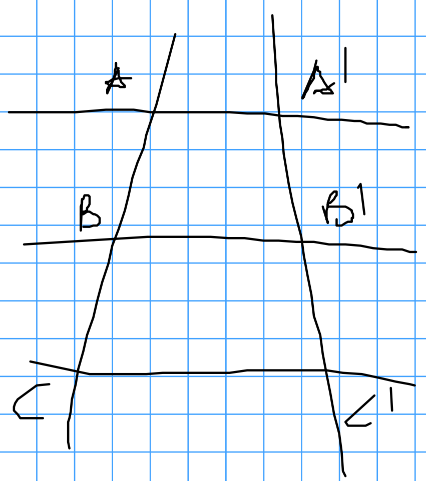
Punti notevoli di un Triangolo

Circonferenze, corde secanti tangenti









$$\frac{AB}{BC} = \frac{A'B'}{B'C'}$$

$\Longrightarrow$ Similitudine dei $\triangle$

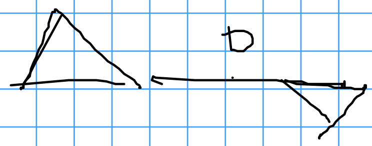
OMOTETIA

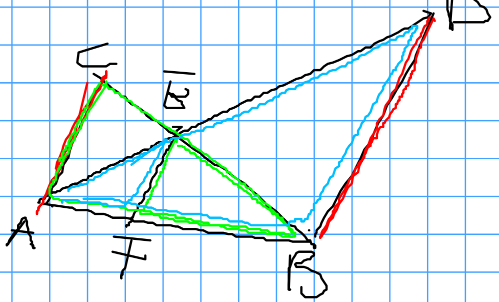
note post-lezione

$k \in \mathbb{R} - \{0\}$

$A' \in OA$

$$\frac{OA'}{OA} = k$$



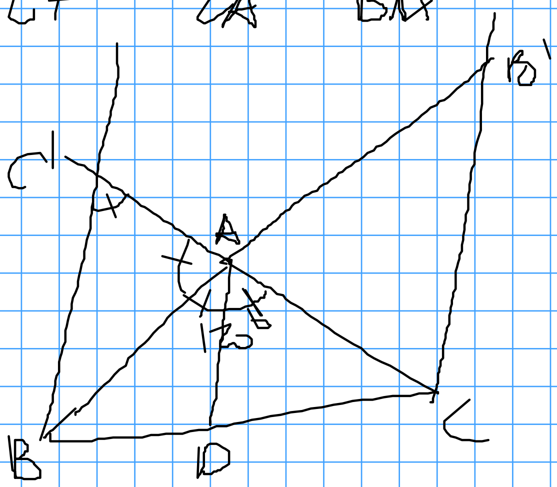


$$\frac{EF}{CA} = \frac{FB}{AB}$$

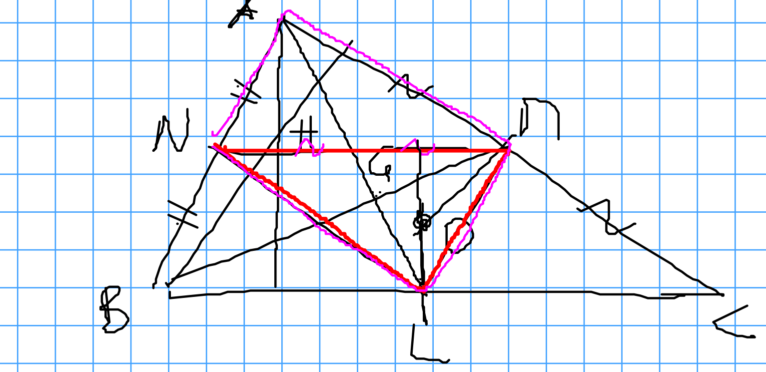
$$\frac{EF}{BD} = \frac{AF}{AB}$$

$$EF\left(\frac{1}{CA} + \frac{1}{BD}\right) = \frac{1}{\cancel{AB}}\left(\cancel{FB+AF}\right)$$

$$\frac{1}{EF} = \frac{1}{CA} + \frac{1}{BD}$$



$$\frac{1}{AD} = \frac{1}{\substack{\cancel{CB'} \\ AB}} + \frac{1}{\substack{C'B \\ AC}}$$



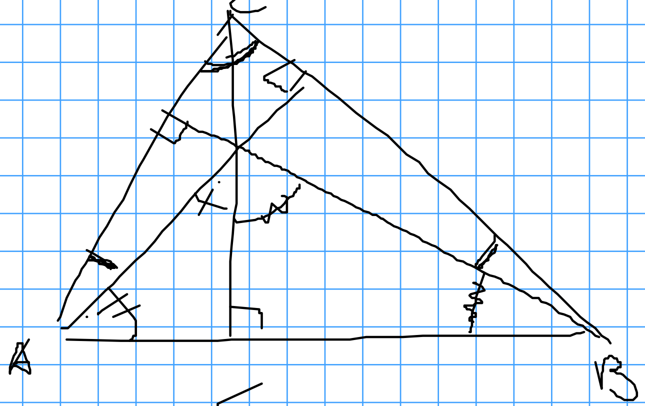
G è bar. sia di ABC che di MNL

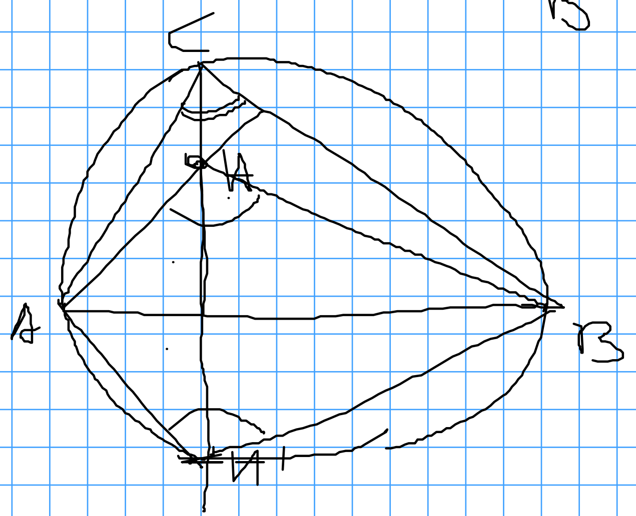
O

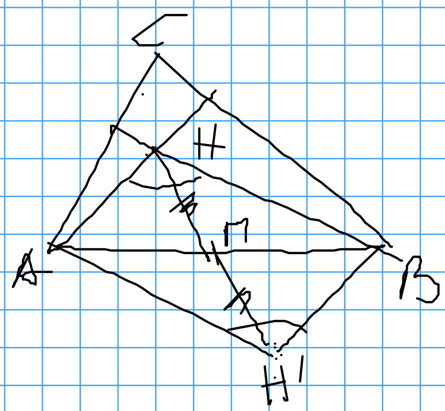
H

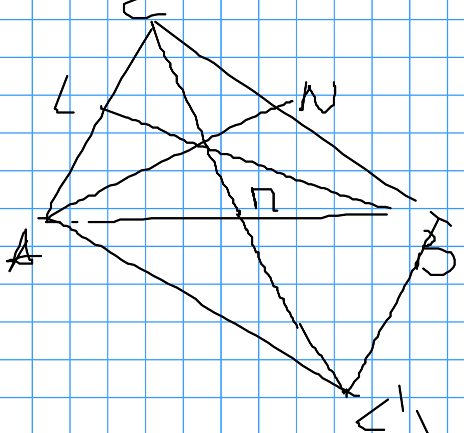
$H_{MNL} \to O_{ABC}$

$G_{MNL} \to G_{ABC}$









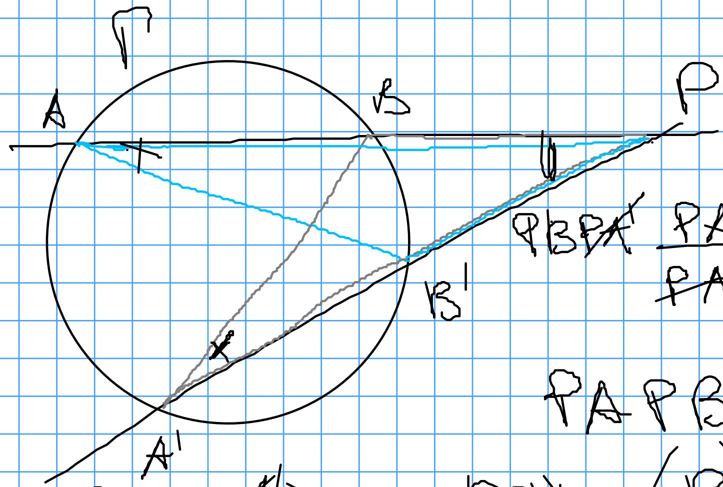
$AB \leq AC + CB$

$2CM \leq AC + CB$

$2AM \leq AB + AC$

$2BL \leq CB + AB$

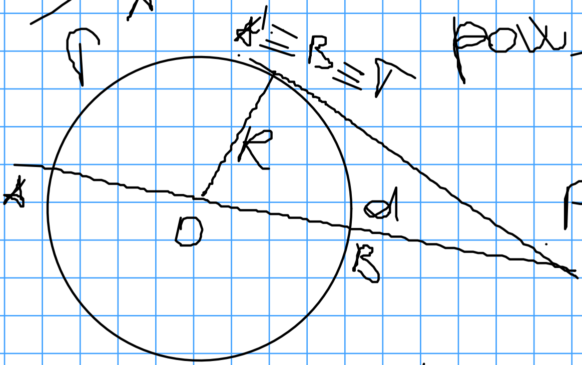
$\cancel{2}(CM + AM + BL) \leq \cancel{2}(2p)$



$\cancel{PB\,PA'}\quad \frac{PA}{\cancel{PA'}} = \frac{PB'}{\cancel{PB}}\quad \cancel{PB\,PA'}$

$PA\,PB = PA'\,PB'$

$pow_\Gamma(P) = PA\,PB$



$\overline{PT}^2$

Γ

$d^2 = \overline{PT}^2 + R^2$ (Pitagora)

$pow_\Gamma(P) = d^2 - R^2$ quantità positiva negativa o nulla

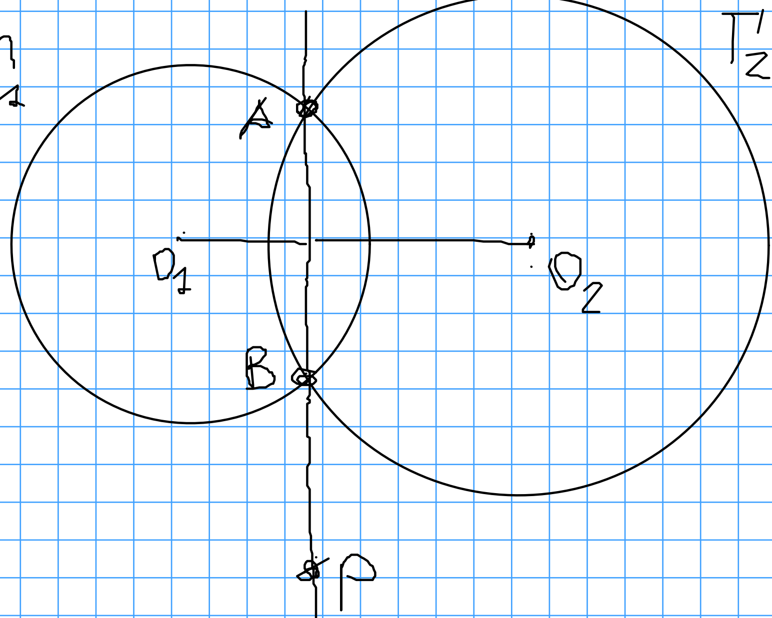
se P è esterno a Γ > 0

$P \in \Gamma$ $= 0$

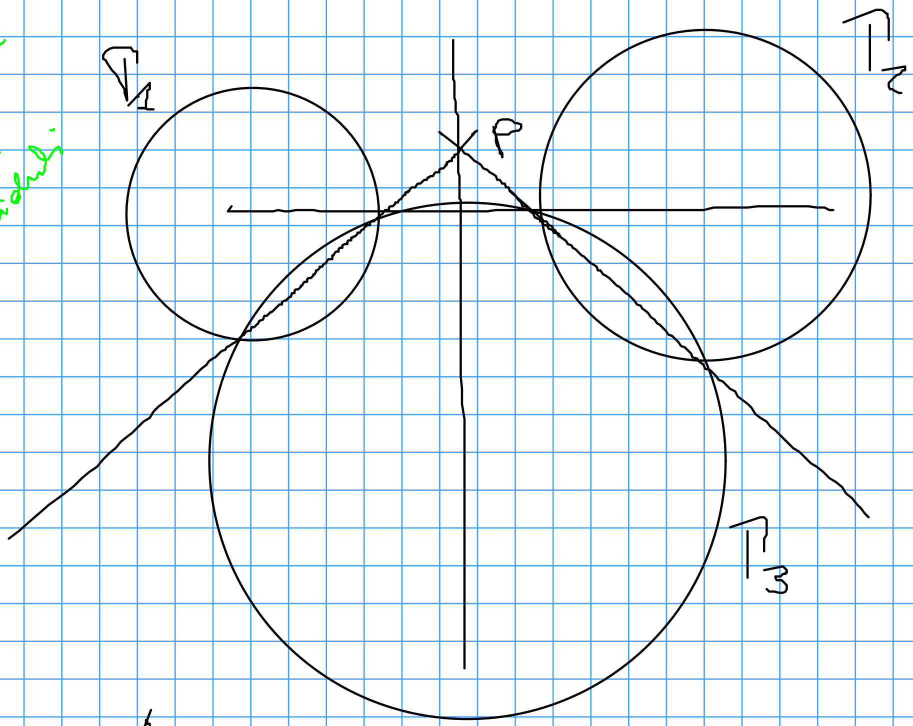
P è interno a Γ < 0

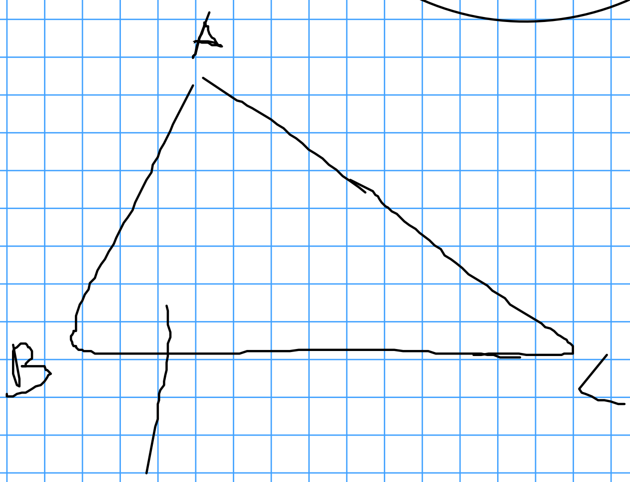
costruzione asse radicale

caso di circ. incidenti



costruire asse radicale caso 2 circonferenze non incidenti





$|pow_\Gamma(I)| = AI \cdot ID$

$$\boxed{AI^2 = r^2 + AH^2}$$

nel caso specifico $pow_\Gamma(I)$ è negativo mentre $\overline{AI} \cdot \overline{ID}$ in quanto prodotto di misure è > 0

$pow_\Gamma(I) = -R^2 + d^2$

$AI \cdot ID = |-R^2 + d^2| = R^2 - d^2$

$$\frac{AI}{DE} = \frac{IH}{BD}$$

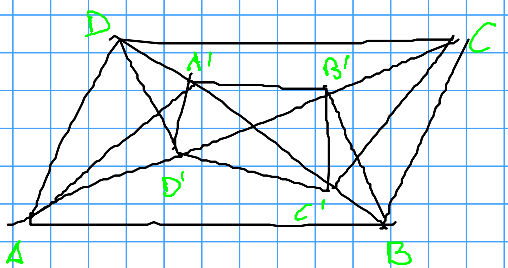
$$\frac{AI}{2R} = \frac{r}{ID}$$

$AI \cdot ID = 2Rr$

$2Rr = |d^2 - R^2|$

$d^2 = R^2 - 2Rr$

esercizi proposti



Dato un parallelogramma ABCD dimostrare che il quadrilatero D'C'B'A' è un parallelogramma simile ad ABCD, dove A', B', C', D' sono le proiezioni di A, B, C, D sulle diagonali AC e BD

dim:

(1) $\triangle ASL \cong \triangle CQL \Rightarrow SL = LQ$;

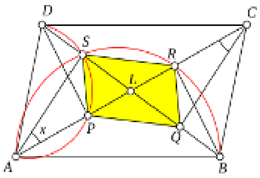
(2) $\triangle DPL \cong \triangle BRL \Rightarrow PL = LR$;

(3) da (1) e (2) segue che

$$\triangle SLP \cong \triangle QLR$$

per cui $PS = QR$ e

$$P\hat{S}L = R\hat{Q}L \Rightarrow PS \parallel QR$$



In modo analogo si prova che $PQ \parallel RS \Rightarrow PQRS$ è un parallelogramma.

Posto $S\hat{A}B = x$, dai quadrilateri ciclici $APSD$, $ASRB$ e dal teorema della corda segue che:

$$\frac{SP}{SR} = \frac{AD \cdot \sin x}{AB \cdot \sin x} = \frac{AD}{AB} \quad (4)$$

$$P\hat{S}R = P\hat{S}Q + Q\hat{S}R = D\hat{A}P + B\hat{A}R = A\hat{D}B \quad (5)$$

$$S\hat{P}Q = 180^\circ - P\hat{S}R = 180^\circ - A\hat{D}B = D\hat{A}B \quad (6)$$

Da (4),(5) e (6) segue che $ABCD \sim PQRS$.

2° problema proposto "in uscita"

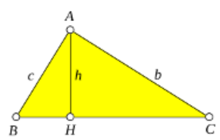
Problema 1. *Dato un triangolo avente i cateti di lunghezza b e c e l'altezza relativa all'ipotenusa lunga h, mostrare che*

$$\frac{1}{h^2} = \frac{1}{b^2} + \frac{1}{c^2}$$

Soluzione.

Indicata con a l'ipotenusa risulta $bc = ah$ ed applicando il teorema di Pitagora abbiamo che:

$$\frac{1}{h^2} = \frac{a^2}{b^2c^2} = \frac{b^2+c^2}{b^2c^2} = \frac{1}{b^2} + \frac{1}{c^2}$$



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