COMBINATORIA

4 primi, 2 secondi, 3 dolci

$4 \cdot 2 \cdot 3$ pasti possibili

Quanti sono i divisori di 6000?

$6000 = 2^4 \cdot 3 \cdot 5^3$

$12 = 2^2 \cdot 3^1 \cdot 5^0$ $\quad 2^a \cdot 3^b \cdot 5^c$

$5 \cdot 2 \cdot 4 = 40$

Quanti sono i possibili ordini di arrivo tra 8 atleti?

| 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

$8 \cdot 7 \cdot 6 \cdot 5 \cdot \ldots \cdot 1 = 8!$

FATTORIALE

40 persone

Quanti modi possiamo scegliere un capitano, un consegnatario, un vice

| 40 | 39 | 38 |

In quanti modi poss. scegliere 2 capitani?

$\frac{40 \cdot 39}{2}$ $\quad$ A, B

B, A

Scegliere 3: $\frac{40 \cdot 39 \cdot 38}{3!}$

ABC
ACB
BAC
BCA
CAB
CBA

3!

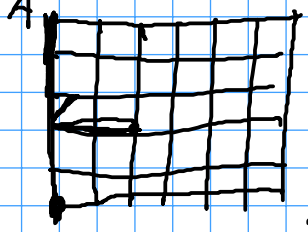
$\binom{40}{3}$ COEFFICIENTE BINOMIALE

$\binom{n}{k}$ = numero di modi di scegliere k persone tra n

$$\frac{n(n-1)\cdot\ldots\cdot(n-k+1)}{k\cdot(k-1)\cdot\ldots\cdot 2\cdot 1}\cdot\frac{(n-k)\cdot(n-k-1)\cdot\ldots 1}{(n-k)!}$$

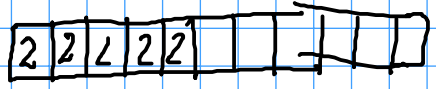
$$= \frac{n!}{k!\,(n-k)!}$$

$$\binom{n}{k} = \binom{n}{n-k}$$



In quanti modi posso andare da A a B?

11 mosse



NON funziona

5 verso il basso, 6 verso destra

$$\binom{11}{5} = \binom{11}{6}$$

In quanti modi si possono scegliere

$$1 \le x_1 < x_2 < x_3 < x_4 < x_5 \le 90 \; ?$$

$$\binom{90}{5}$$

In quanti

$$1 \le x_1 \le x_2 \le x_3 \le x_4 \le x_5 \le 30 \quad ?$$

$y_1 = x_1$
$y_2 = x_2 + 1$
$y_3 = x_3 + 2$
$y_4 = x_4 + 3$
$y_5 = x_5 + 4$

$$1 \le y_1 < y_2 < y_3 < y_4 < y_5 \le 34$$

$$\binom{34}{5}$$

ANAGRAMMI

Quanti sono gli anagrammi di ...

- CASO $\boxed{4}\boxed{3}\boxed{2}\boxed{1} = 4!$
- CASÅ CAÅS CÅSA CÅAS $\quad \frac{4!}{2}$
- PASSÅTÅ $\quad \frac{7!}{2! \cdot 3!}$
  SS

Con quanti zeri termina $35!$?

$35! = 2^{\boxed{a}} 3^{b} 5^{\boxed{c}} \cdots$

$= 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdots$

$\left(\left[\frac{35}{2}\right] = 17\right) + \left[\frac{35}{4}\right] = 8 \quad \left[\frac{35}{8}\right] = 4$

$+ \left[\frac{35}{16}\right] = 2 \quad + \left[\frac{35}{32}\right] = 1$

$\left[\frac{35}{5}\right] = 7 \qquad \left[\frac{35}{25}\right] = 1 \qquad$ 8 fattori 5

Termina con 8 zeri

Probabilità $= \dfrac{\#\text{ casi favorevoli}}{\#\text{ casi possibili}}$

Probabilità che 5 numeri estratti da 1 e 90 siano in ordine crescente?

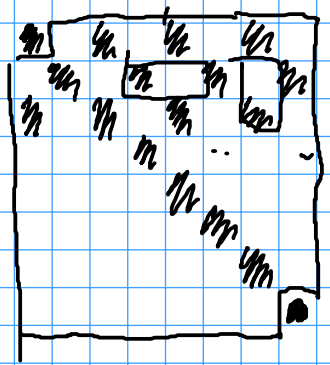
casi favorevoli : $\binom{90}{5}$

casi possibili : $90 \cdot 89 \cdot 88 \cdot 87 \cdot 86$

$$p = \frac{1}{5!}$$

COLORAZIONI

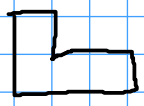
Scacchiera 8×8 senza 2 quadretti opposti.
È possibile coprirla con pezzi ▭ ?



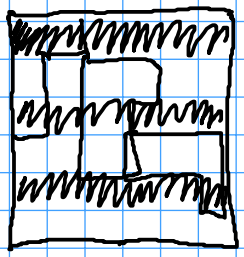
62

▭

No, non è possibile

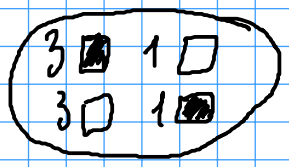
6×6  Si riesce?

6×8



18 bianchi

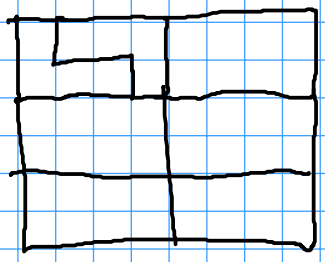
18 neri



9 pezzi a L

$3+1+1+3+1+3+1+1+3=18$

9

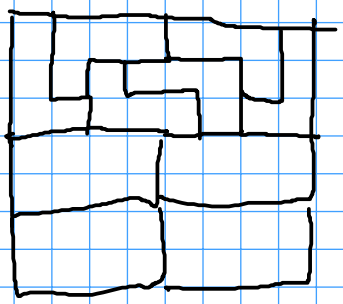


6×8

2×4

a×8 b

3×8



ESERCIZI

① $n = 2^a \cdot 3^b \cdot 5^c \cdot \dots$

$(a+1)(b+1)(c+1)\cdot\dots$ quadrati

$a, b, c, \dots$ pari

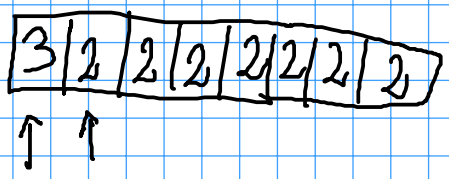
24

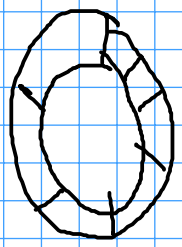
1	24
2	12
3	8
4	6
6	4
⋮	⋮

36

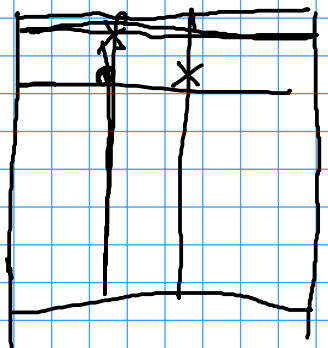
1	36
2	18
3	12
4	9
6	6

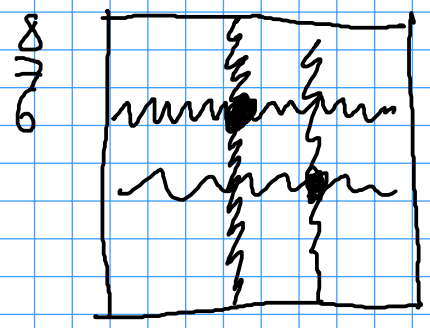
2)





3) $8!$





$64 \cdot 49 \cdot 36 \cdot$

$$\frac{(8!)^2}{8!} = 8!$$

4) $1 \le x_1 \le x_2 \dots \le x_5 \le 30$

$1 \le x_1 < x_2 - 1 < \dots$ $\qquad y_{11} = x_{11} - 10$

$y_1 = x_1, \quad y_2 = x_2 - 1, \quad y_3 = x_3 - 2$

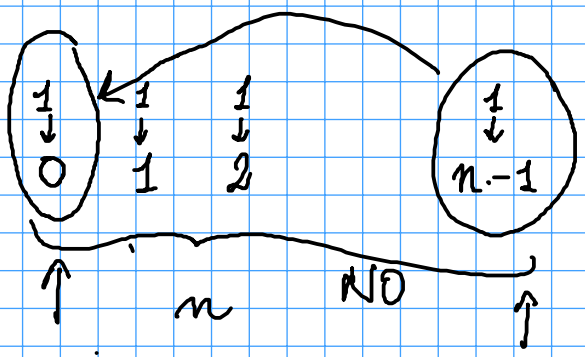
$1 \le y_1 < y_2 < y_3 < \dots < y_{11} \le 16 \qquad \binom{16}{11}$

5) Principio dei cassetti

$n+1$ calzini $\rightarrow$ n cassetti

In Almeno un cassetto ci sono almeno 2 calzini.

n persone



6) Contano la stessa cosa.

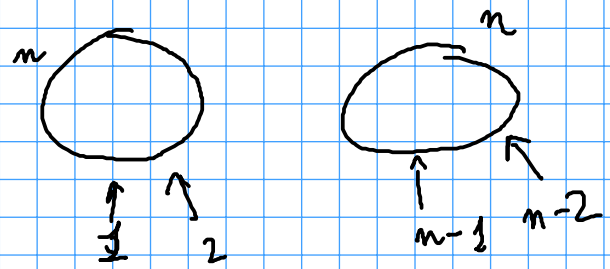
$\binom{2n}{n}$ = modi di scegliere n persone tra 2n

$0! = 1$

$\binom{n}{0} = \binom{n}{n}$ $\quad \binom{n}{1} = \binom{n}{n-1}$ $\quad \binom{n}{k} = \binom{n}{n-k}$

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots + \binom{n}{n}^2 =$$

$$= \binom{n}{0}\binom{n}{n} + \binom{n}{1}\binom{n}{n-1} + \dots + \binom{n}{k}\binom{n}{n-k} + \dots +$$



Scegliere n persone tra 2n

7) SENZA $5!$

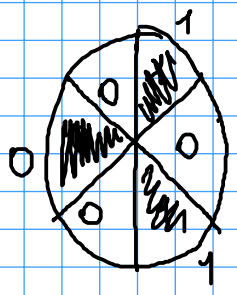
SASSO $20 = \frac{5!}{3!}$

8) casi possibili $\binom{90}{6}$

casi favorevoli $\binom{89}{5}$

$$\frac{\binom{89}{5}}{\binom{90}{6}} = \frac{\frac{89 \cdot 88 \cdot 87 \cdot 86 \cdot 85}{5!}}{\frac{90 \cdot 89 \cdot 88 \cdot 87 \cdot 86 \cdot 85}{6 \cdot 5!}} = \frac{6}{90}$$

9)



Ad ogni mossa

$S_N - S_B = 2$