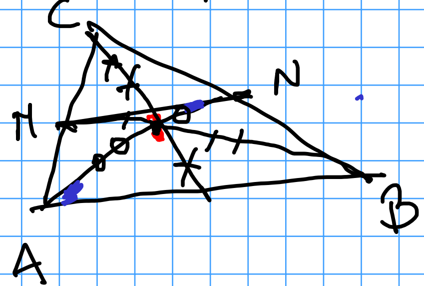
# GEOMETRIA

## Triangoli

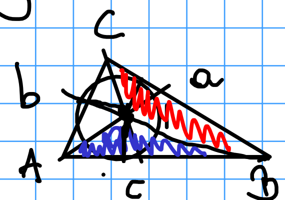
Criteri di similitudine:

Congruenza

BARICENTRO: punto di incontro delle mediane




INCENTRO




$r =$

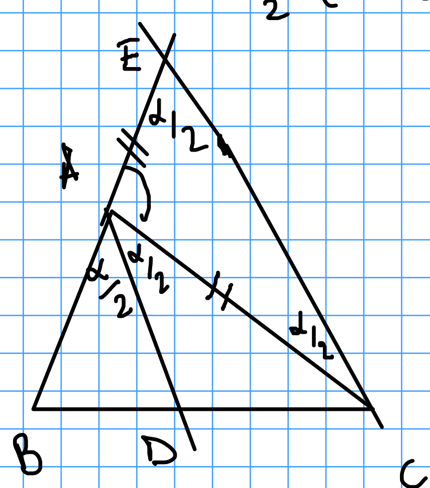
$= r \cdot c/2$

$= \frac{r \cdot a}{2}$

$= \frac{r b}{2}$

$2p = a+b+c$

$S = \frac{r}{2}(a+b+c)$ $\qquad r = \frac{S}{p}$




$BD : DC = AB : AC$

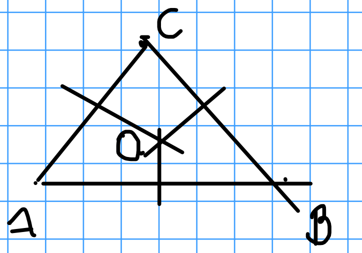
$\overset{\triangle}{ABD} \sim \overset{\triangle}{EBC}$

$BD : BC = AB : EA$

$\| $

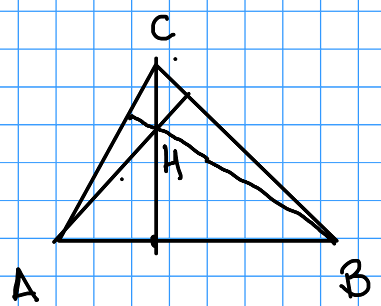
$AC$

CIRCOCENTRO



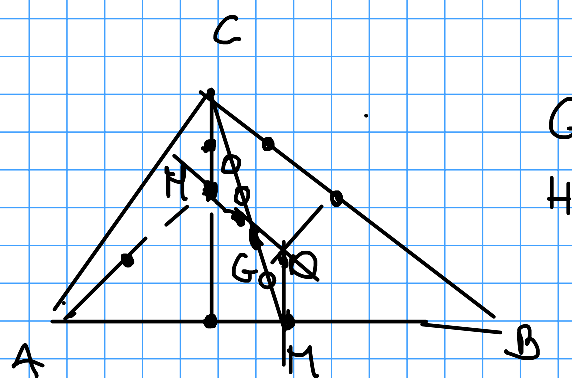

$$R = \frac{abc}{4S}$$

ORTOCENTRO




B è ortocentro di AHC

Retta di Eulero: H G O $HG = 2GO$

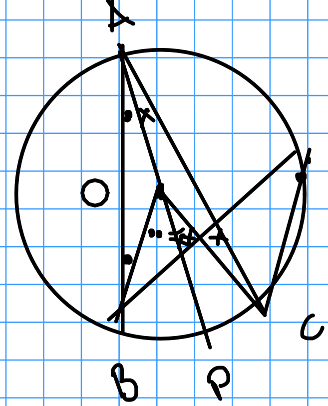



$G\overset{\triangle}{H}C \sim G\overset{\triangle}{O}M$

$HG = 2\,GO$

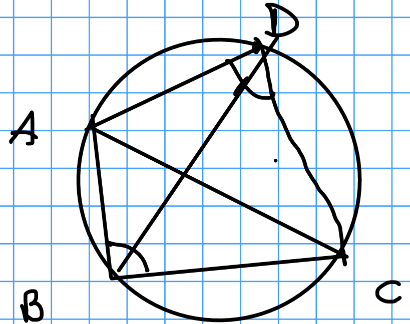
Circonferenza di Feuerbach

CIRCONFERENZE



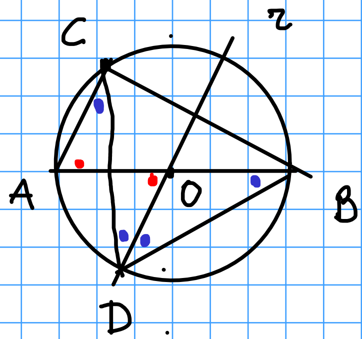

AOB

Quadrilateri ciclici




angoli opposti supplementari

$AC \cdot BD = AD \cdot BC + AB \cdot CD$ (Tolomeo)



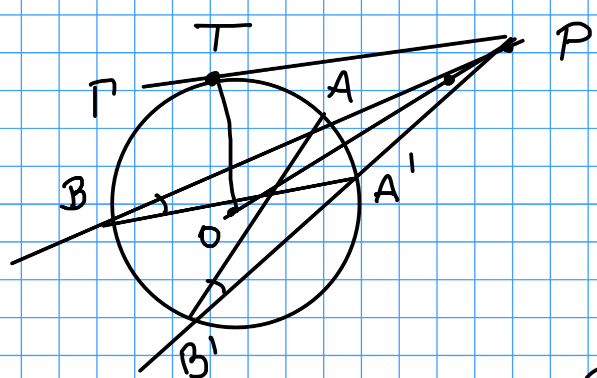

$r \parallel AC$

1) $DO$ bisettrice di $C\hat{D}B$

2) $\overset{\triangle}{CDB}$ simile ad $\overset{\triangle}{AOD}$

$A\hat{O}D = C\hat{D}B$ + entrambi isosceli quindi sono simili.

## Potenza di un punto rispetto a una circonferenza



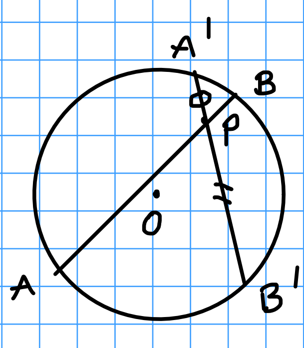

$$Pot_\Gamma(P) = PA \cdot PB = PT^2$$

$$PA' \cdot PB' = PA \cdot PB$$

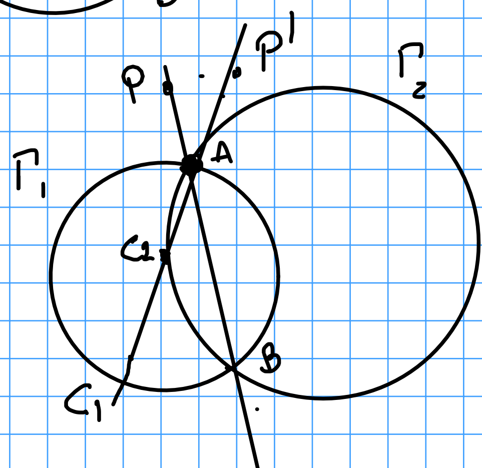
$$\overset{\triangle}{PA'B} \sim \overset{\triangle}{PAB'}$$

$$Pot_\Gamma(P) = PT^2 = d^2 - r^2$$

$\overset{\triangle}{POT}$ rettangolo



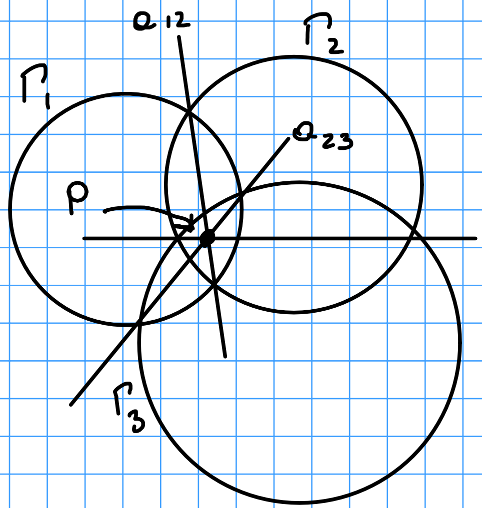

$$Pot_\Gamma(P) = -PA \cdot PB$$
$$= -PA' \cdot PB'$$




Quali sono i punti $P$ per cui $Pot_{\Gamma_1}(P) = Pot_{\Gamma_2}(P)$?

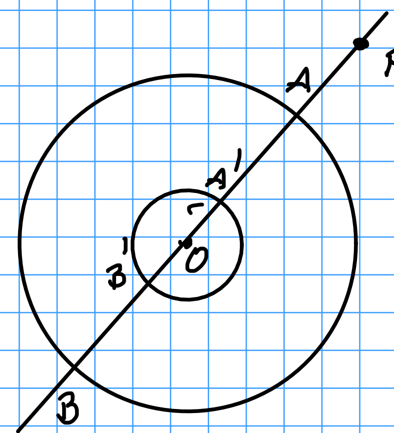
$$Pot_{\Gamma_1}(P) = Pot_{\Gamma_2}(P) = PA \cdot PB$$

$Pot_{\Gamma_2}(P) = PA \cdot PC_2 \neq PA \cdot PC_1 = Pot_{\Gamma_1}(P)$




$a_{12}$ $\quad a_{23}$

$$Pot_{\Gamma_1}(P) = Pot_{\Gamma_2}(P) = Pot_{\Gamma_3}(P)$$




$PA \cdot PB < PA' \cdot PB'$

$(PA, PB)$

$(PA', PB')$

NUMERI POSITIVI

Media aritmetica $\quad \dfrac{a_1 + a_2}{2} = MA$

Media geometrica $\quad \sqrt{a_1 \cdot a_2} = MG$

$MA \geq MG$ $\quad a_1 + a_2 \overset{?}{\geq} 2\sqrt{a_1 a_2}$

$$a_1^2 + 2a_1a_2 + a_2^2 \overset{?}{\geq} 4a_1a_2$$

$$a_1^2 - 2a_1a_2 + a_2^2 \overset{?}{\geq} 0$$

$$(a_1 - a_2)^2 \geq 0$$

$a_1, a_2 \geq 0$ $\quad \dfrac{a_1 + a_2}{2} = m$

Qual è il massimo valore che può assumere $a_1 \cdot a_2$ con ?

$$a_1 \cdot a_2 = (GM)^2 \leq m^2$$

$$a_1 = m \,, \quad a_2 = m$$

$a_1, a_2, a_3, \dots, a_n$

$$MA = \frac{a_1 + a_2 + \dots + a_n}{n}$$

$$MG = \sqrt[n]{a_1 a_2 \cdot \dots \cdot a_n}$$

Esercizio: $x + y = 1 \qquad x, y \geq 0$

Quanto può valere al massimo $x^2 y$ ?

$\frac{x}{2}, \frac{x}{2}, y$

$x^2 y \leq \dots$

$$\frac{\frac{x}{2} + \frac{x}{2} + y}{3} = \frac{1}{3} = MA$$

$$\sqrt[3]{\frac{x^2 y}{4}} = MG$$

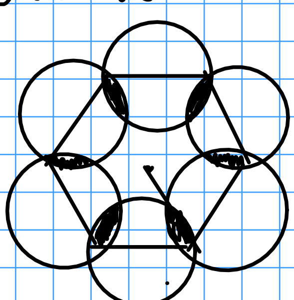
$MG \leq MA$

$$\frac{x^2 y}{4} \leq \frac{1}{27}$$

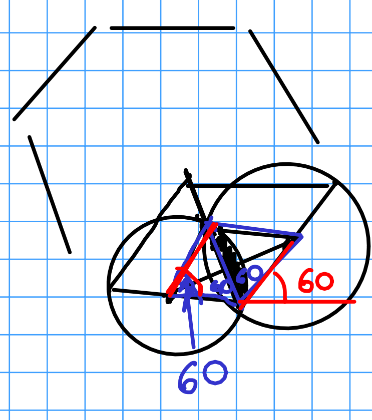
$$\boxed{x^2 y \leq \frac{4}{27}}$$

$$\frac{x}{2} = y \qquad x = \frac{2}{3} \,, \; y = \frac{1}{3} \qquad x^2 y = \frac{4}{27}$$

Esercizio 1

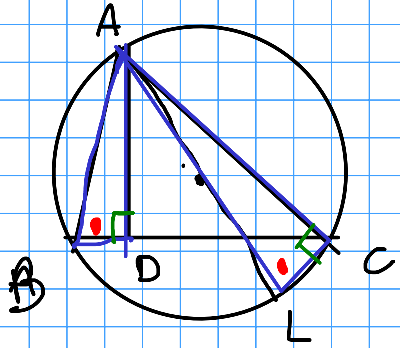


$6(\pi r^2 - \quad)$




## Es. 2

$$R = \frac{abc}{4S} \qquad \boxed{S = \frac{abc}{4R}}$$



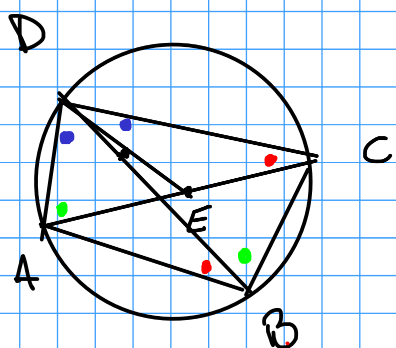

$$S = \frac{a \cdot h_a}{2} \qquad h_a = \frac{bc}{2R}$$

$$h_a : AB = AC : AL$$

$$\phantom{h_a :}\ c \qquad b \qquad 2R$$

## Es 3 Dimostrare Tolomeo

$$AD \cdot BC + AB \cdot DC \geq AC \cdot BD$$




1) $\overset{\triangle}{CDE} \sim \overset{\triangle}{BDA}$ ✓

2) $\overset{\triangle}{ADE} \sim \overset{\triangle}{BDC}$ ✓

$EC : BA = CD : BD$

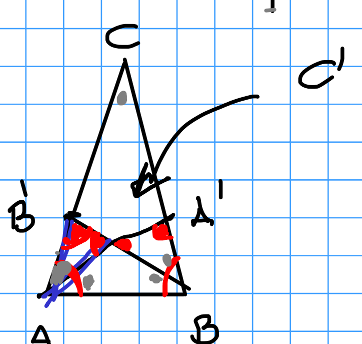
$$AC = AE + EC$$

$$AC \cdot BD = AE \cdot BD + EC \cdot BD$$

$$AE : BC = AD : BD$$

$$= BC \cdot AD + BA \cdot CD$$

## Esercizio 4




$ABB'$ isoscele — $B\hat{A}C' = A\hat{C}B$

$AB'C'$

$\overset{\triangle}{AA'B} \sim \overset{\triangle}{CAB}$

$5 \bullet = 180°$

$\bullet = 36°$