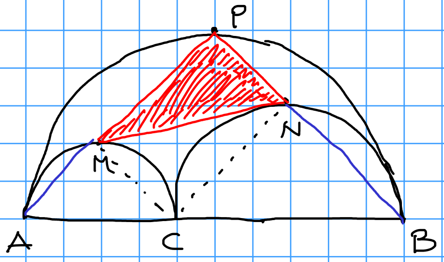
GEOMETRIA - ESERCIZI.

ESERCIZIO 1

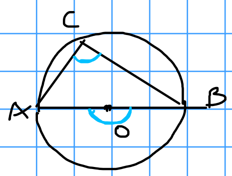


$AB = 4$ cm
$AC = 1$ cm

Area $(PMN) = ? = \frac{3}{4}$ cm^2.

1° osservazione: $A\hat{M}C$, $C\hat{N}B$ e $A\hat{P}B$ sono isosceli e rettangoli

(triangoli inscritti in semicirconferenze



$A\hat{C}B = \frac{180°}{2} = 90°$)

$M\hat{A}C = 45° = P\hat{A}C \Rightarrow A, M, P$ sono allineati
Allo stesso modo $\Rightarrow P, N, B$ sono allineati
- $A\hat{P}B = M\hat{P}N = 90°$

$$\text{Area }(PMN) = \frac{PM \cdot PN}{2}$$

$PM = PA - AM$ $\qquad$ $PN = PB - BN$

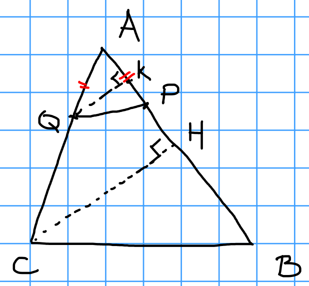
$AB = 4$ cm $\qquad$ $AC = 1$ cm $\qquad$ $AM = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ cm

$BN = \frac{3}{2}\sqrt{2}$ cm

$PA = PB = \frac{4}{2}\sqrt{2} = 2\sqrt{2}$ cm

$$\text{Area} = \left(2\sqrt{2} - \frac{\sqrt{2}}{2}\right)\left(2\sqrt{2} - \frac{3}{2}\sqrt{2}\right) \cdot \frac{1}{2} = \underbrace{\frac{3\sqrt{2} \cdot \sqrt{2}}{8}}_{} = \frac{3}{4} \text{ cm}^2$$

Esercizio 2



$AB = 60\ cm \qquad AC = 50\ cm \qquad Area\ (ABC) = 720\ cm^2$

$AQ = AP = 10\ cm$

$Area\ (AQP) = ?$

$$Area\ (AQP) = \frac{AP \cdot QK}{2} = 5 \cdot QK$$

$$720 = \frac{AB \cdot CH}{2} = 30 \cdot CH \rightarrow CH = 24\ cm$$

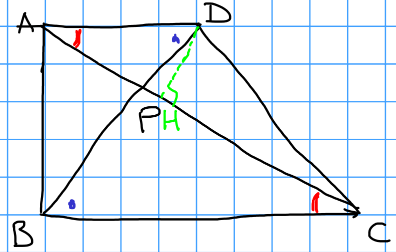
Consideriamo $A\hat{Q}K$ e $C\hat{A}H$: sono simili :

- $\hat{H} = \hat{K} = 90°$
- $Q\hat{A}K$ è in comune

$$\frac{QK}{CH} = \frac{AQ}{AC} \qquad \frac{QK}{24} = \frac{10}{50} \rightarrow QK = \frac{24}{5}\ cm$$

$$Area\ (AQP) = 5 \cdot QK = 24\ cm^2$$

ESERCIZIO 3



Area $(APD) = 396\ cm^2$
Area $(BPC) = 539\ cm^2$

Area $(ABCD) = ?$

Consideriamo $A\hat{P}D$ e $B\hat{P}C$: sono simili :

- $A\hat{P}D = B\hat{P}C$ perché opposti al vertice
- $D\hat{A}P \cong P\hat{C}B$ perché alterni interni in $BC // AD$ tagliate da AC

$$\frac{AP}{PC} = \frac{PD}{PB} = k$$

$$\frac{\text{Area }(APD)}{\text{Area }(BPC)} = k^2 = \frac{396}{539} = \frac{36}{49} \qquad k = \frac{6}{7}$$

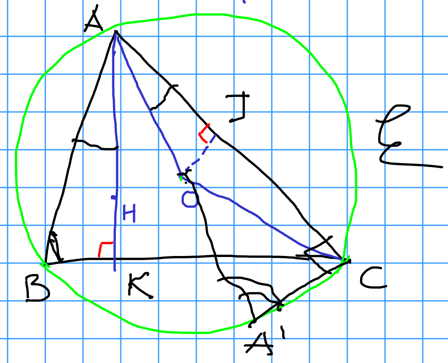
$$AP = \frac{6}{7} PC \qquad PD = \frac{6}{7} PB$$

$$\text{Area }(DPC) = ? = \frac{PC \cdot DH}{2} = \frac{7}{6} AP \cdot \frac{DH}{2} = \frac{7}{6}\left(\frac{AP \cdot DH}{2}\right) = \frac{7}{6} \cdot 396 = 462 \quad (1)$$

$$\text{Area }(APB) = \frac{7}{6}\text{ Area }(CPD) = \frac{7}{6} \cdot 396 = .$$

$$\text{Area }(ABCD) = 396 + \frac{7}{6} \cdot 396 + \frac{7}{6} \cdot 396 + 539 = 1859\ cm^2$$

ESERCIZIO 4



Vogliamo $\overset{\triangle}{BAK} \sim \overset{\triangle}{OAJ}$

$B\hat{K}A \cong O\hat{J}A$ retti

$A\hat{B}C$ insiste su $\overline{AC}$

$A\hat{O}J \cong \frac{1}{2} A\hat{O}C \cong A\hat{B}C$

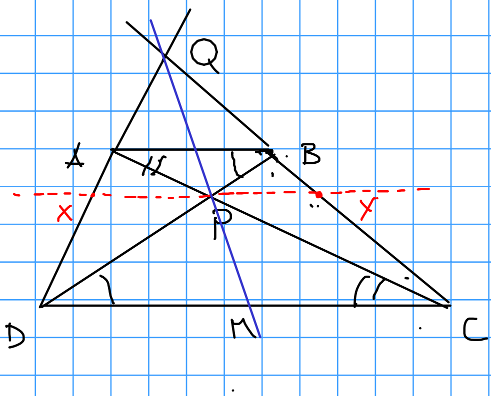
$\Rightarrow B\hat{A}K \cong O\hat{A}J$

Altro modo: $A' \equiv AO \cap \mathcal{C} \Rightarrow AA'$ diametro

$\Rightarrow A\hat{C}A' \cong A\hat{K}B$. $A\hat{B}K \cong A\hat{A'}C$ perché insistono su $\overline{AC}$.

Quindi Tesi.

ESERCIZIO 5



$AB \parallel XY \parallel CD$

TESI: • $PX = PY$
• $DM = MC$

Tattica: $\cancel{\triangle APB \sim \triangle PDC}$
• $\triangle DXP \sim \triangle DAB$ e $\triangle CPY \sim \triangle CAB$

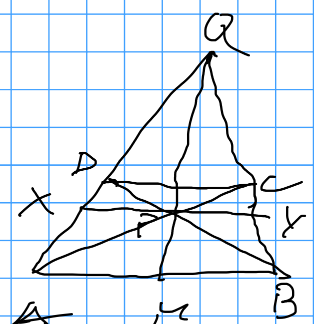
$\frac{XP}{AB} = \frac{DP}{DB}$; $\frac{YP}{AB} = \frac{CP}{PA}$

Ci basta $\frac{DP}{DB} = \frac{CP}{CA} \iff DP \cdot CA = CP \cdot DB$

$\iff DP(CP + PA) = CP(DP + PB) \iff DP \cdot PA = CP \cdot PB$

$\iff \frac{DP}{CP} = \frac{PB}{PA}$ è vero.

2.



$\triangle XQP \sim \triangle AQM$

$\triangle YQP \sim \triangle BQM$

$\frac{XP}{AM} = \frac{QP}{QM}$; $\frac{YP}{MB} = \frac{QP}{QM}$

$\Rightarrow \frac{\cancel{XP}}{AM} = \frac{\cancel{YP}}{MB} \Rightarrow AM = MB$ □