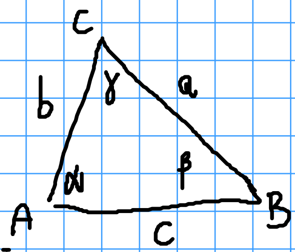
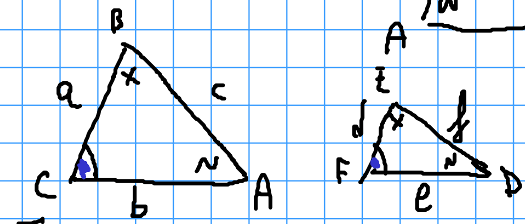
GEOMETRIA

9/2/2018

SIMILITUDINI



$ABC \sim DEF$



$a:d = b:e = c:f$

I) 3 angoli uguali (bastano 2)

II) 2 coppie di lati direttamente proporzionali e l'angolo compreso congruente

III) 3 coppie di lati direttamente proporzionali

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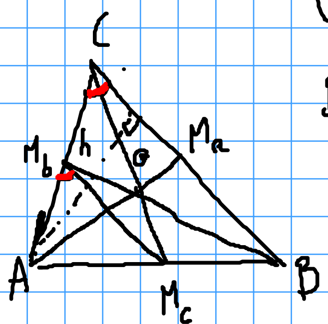
Punti Notevoli di un triangolo

Baricentro (mediane) G



$BM_a = CM_a$, $CM_b = AM_b$, $AM_c = BM_c$

$AG = 2GM_a$

Area di ABM_a = Area di $ACM_a = [ABM_a]$

$M_bM_c // BC$

AM_bM_c e ACB. L'angolo in A é in comune

$AM_b : AC = AM_c : AB = 1/2$

$M_bM_c = CM_a = BM_a = \frac{1}{2} BC$

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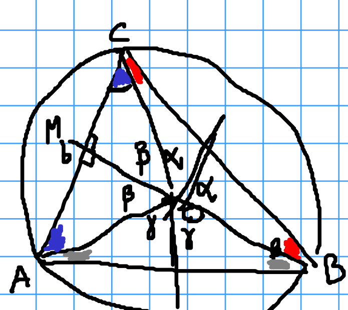
Circocentro (assi) O



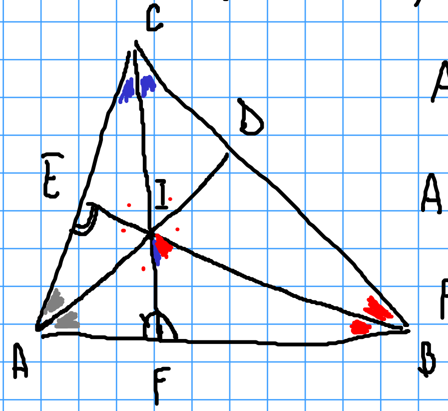
$AO = OC = OB$

$A\hat{O}C = 2A\hat{B}C = 2\beta$

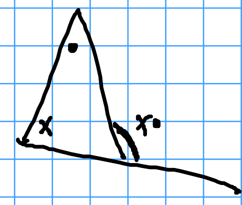
$A\hat{C}O = 90° - \beta$

$O\hat{C}B = 90 - \alpha$

Incentro (bisettrici) I



$A\hat{C}I = I\hat{C}B = \gamma/2$



$A\hat{F}C = \gamma/2 + \beta$

$$F\hat{I}B = 180° - \underset{\alpha + \gamma/2}{I\hat{F}B} - \underset{\beta/2}{I\hat{B}F} = 180 - \frac{\alpha}{2} - \overbrace{\frac{\alpha}{2} - \frac{\beta}{2} - \frac{\gamma}{2}}^{90} =$$

$\alpha + \beta + \gamma = 180$

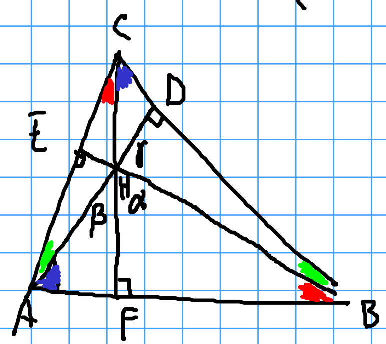
$90 = \frac{\alpha + \beta + \gamma}{2}$

$$= 180 - 90 - \frac{\alpha}{2} = 90 - \frac{\alpha}{2} = \frac{\beta + \gamma}{2}$$

Teorema delle Bisettrici

$AC : AF = BC : BF$

Ortocentro (altezze) H

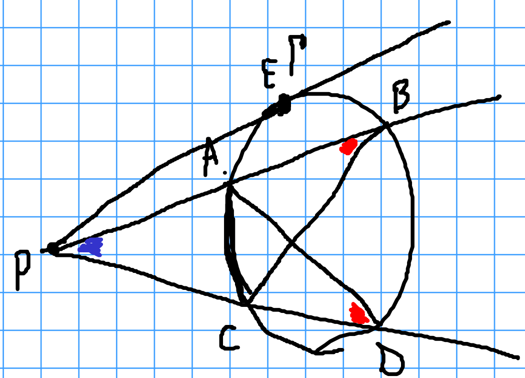


$= 90 - \beta$

$= 90 - \alpha$

$= 90 - \gamma$

Potenza di un punto rispetto ad una circonferenza



$pow_{\Gamma} P = PA \cdot PB = PC \cdot PD = PE^2$

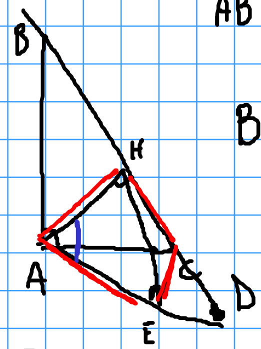
$PAD \sim PBC$

$PA : PC = PD : PB$

$PA \cdot PB = PC \cdot PD$

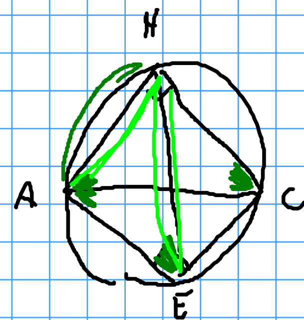
Esercizi (2005)

AB > AC



BH = HD

Tesi: EH = AH



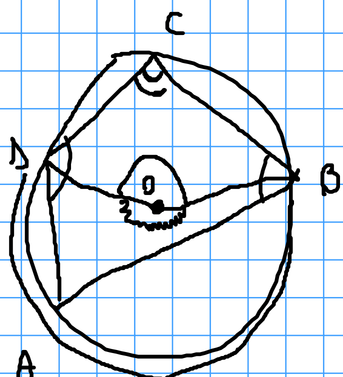
Soluzione:

ABD é isoscele (altezza e mediana coincidono)

$A\hat{D}H = \beta$. $H\hat{A}E = \gamma$

Tesi: $H\hat{E}A = \gamma$. ←

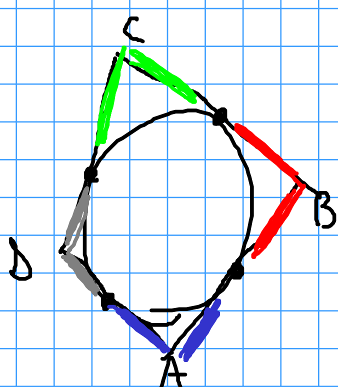
AECH é ciclico (inscrittibile in una circonferenza)



$D\hat{O}B = 2D\hat{A}B$

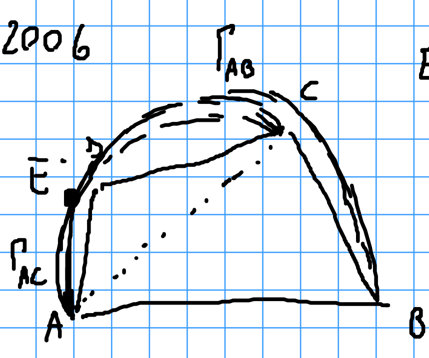
$360 - D\hat{O}B = \underline{360 - 2D\hat{A}B = 2D\hat{C}B}$

$180 - \cancel{D\hat{A}B} = D\hat{C}B + D\hat{A}B$



$AB + CD = | + | + | + | = BC + AD$

2006

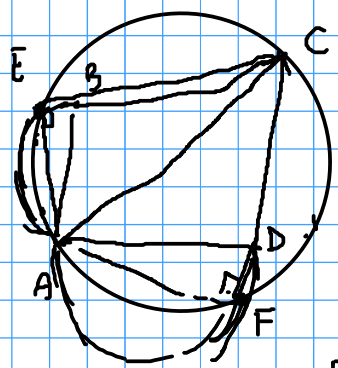


$E = \Gamma_{AB} \cap \Gamma_{AC}$

Tesi:

a) se $E\hat{A}D = 90°$ allora $BC \parallel AD$

$$\left.\begin{array}{l} A\hat{E}C = 90° \Rightarrow EC \parallel AD \\ A\hat{E}B = 90° \Rightarrow EB \parallel AD \end{array}\right\} \Rightarrow BC \parallel AD$$



b) se $E\hat{A}D = F\hat{A}B = 90°$ allora $ABCD$ é un parallelogramma

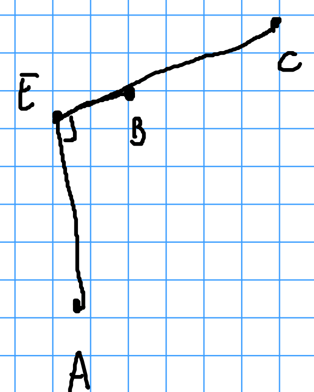
$$\left.\begin{array}{l} F\hat{A}B = 90° \Rightarrow AB \parallel DC \\ E\hat{A}D = 90° \Rightarrow BC \parallel AD \end{array}\right\} \Rightarrow ABCD \text{ parallelogramma}$$

c) Se $ABCD$ é un ▱ allora $E\hat{A}D = F\hat{A}B = 90°$

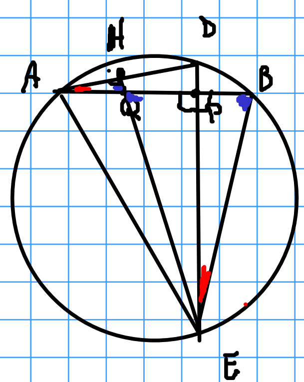
E, B, C allineati

F, D, C allineati

$C\hat{F}A = C\hat{E}A = 90°$



2008



$AP = 2PB$

$AQ = \frac{1}{2} AP$

Tesi:

$Q =$ ortocentro di ADE

Tesi: $EH \perp AD$

QEB isoscele (PE é sia altezza sia mediana)

$E\hat{Q}B = E\hat{B}Q = A\hat{Q}H$

$D\hat{A}B = D\hat{E}B$

$AHQ \sim BEP \Rightarrow A\hat{H}Q = A\hat{H}E = 90°$.

Esercizio 2: Dimostrare che le bisettrici di $A\hat{S}T$ e $B\hat{T}S$ si incontrano ......

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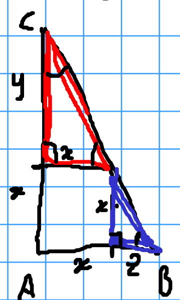
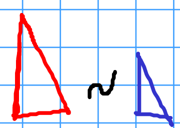
Esercizio 1. Tesi Cesenatico 2012 - 1



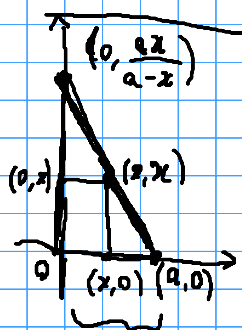
$$\frac{1}{x} = \boxed{\frac{1}{AB} + \frac{1}{AC}}$$

$$\frac{AB \cdot AC}{AB + AC} = x\cancel{(AB+AC)}$$



$$y : x = x : z \Rightarrow \underline{yz = x^2}$$

$$\frac{AB \cdot AC}{AB+AC} = \frac{(x+z)(x+y)}{2x+y+z} = \frac{x^2+xy+xz+\overset{x^2}{\overset{\shortparallel}{yz}}}{2x+y+z} = \frac{2x^2+xy+xz}{2x+y+z} = x$$



$$y = mX + q$$

$$\begin{cases} 0 = am + q \\ x = mx + q \end{cases} \Rightarrow \begin{cases} 0 = am + q \Rightarrow q = -am = -\frac{ax}{x-a} \\ \frac{x}{x-a} = m\cancel{(x-a)} \end{cases}$$

$$y = \frac{x}{x-a}X - \frac{ax}{x-a}$$

$$\frac{1}{AB} + \frac{1}{AC} = \frac{1\cancel{x}}{a\cancel{x}} + \frac{a-\cancel{x}}{ax} = \frac{a}{ax} = \frac{1}{x}$$

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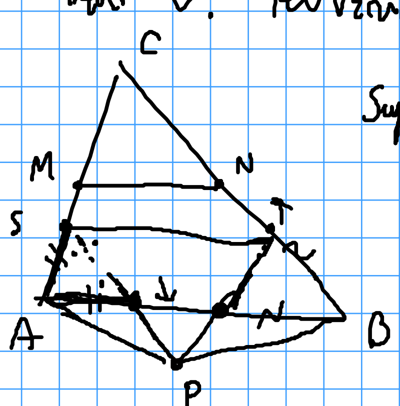
Esercizio 2. Febbraio 2002



Supponiamo AMNB circoscrittibile

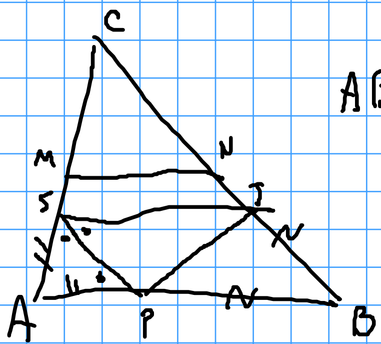
$$\underline{AM + NB} = AB + MN = \frac{3}{2} AB$$

$$3AB = AC + BC$$

$$AB = \frac{AC}{3} + \frac{BC}{3} = AS + BT$$

$$/\!/ + \mathcal{N} = AB$$

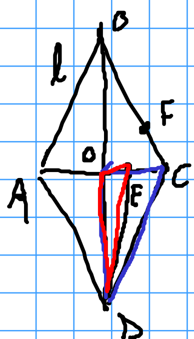
P sta su AB



$$AB + MN = \frac{3}{2}AB = \frac{3}{2}(AP + PB) = \frac{3\,AS + 3\,BT}{2} =$$

$$= \frac{AC}{2} + \frac{BC}{2} = AM + BN$$

Esercizio 4. Febbraio 2006



Tesi: $(AB + BF) \cdot FC = AE \cdot EC$

Soluzione: $AB = l$, $BF = a$, $FC = l - a$

$$(AB+BF) \cdot FC = (l+a)(l-a) = l^2 - a^2 =$$

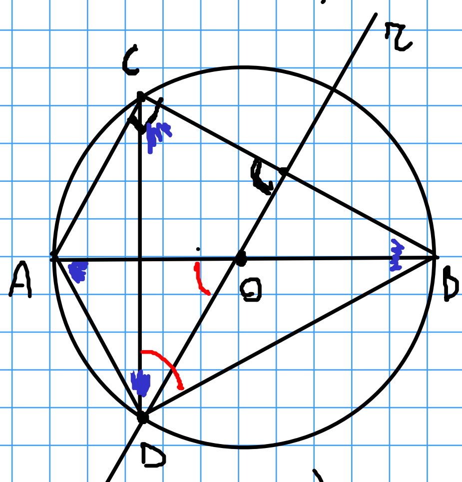
$\rightarrow l^2 = OD^2 + OC^2$

$\rightarrow a^2 = OD^2 + OE^2$

$$= OC^2 - OE^2 = (\underbrace{OC}_{AO} + OE)(OC - OE) =$$

$$= AE \cdot EC.$$

Esercizio 5. Febbraio 2007



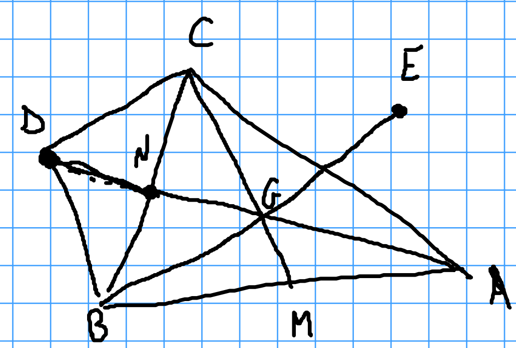
Tesi: 1) DO bisettrice di $C\hat{D}B$

r è asse di BC; è anche altezza, quindi è anche <u>bisettrice</u> e mediana

2) $CDB \sim AOD$

Sono entrambi isosceli e $B\hat{A}D = D\hat{C}B$. FINE

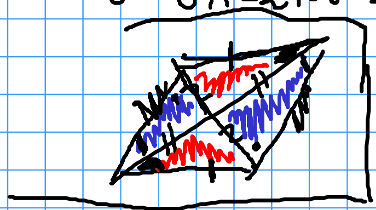
Esercizio 6. Cesenatico 2007, 3.



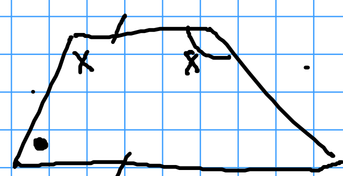
Tesi: BMCD ciclico se e solo se $BA = BE$.

Soluzione:

$DG = GA = 2NG \Rightarrow N$ punto medio di DG



BMCD trapezio



BMCD è ciclico se e solo se è isoscele se e solo se $BM = CD$, ma $CD = BG$, quindi $BM = CD$ se e solo se $BG = BM$ se e solo se $AB = BE$.