COMBINATORIA

31/1/19

Conteggi

Menù

3 antipasti, 5 primi, 3 secondi, 2 dolci

$a_1\ a_2\ a_3 \quad p_1 \dots p_5 \quad s_1\ s_2\ s_3 \quad d_1\ d_2$

$\underbrace{a_1 p_1 s_1}\ d_1 - a_1 p_1 s_1 d_2 - a_1 p_1 s_2 d_1 - a_1 p_1 s_2 d_2$

$a_1 p_1 s_3 d_1 - a_1 p_1 s_3 d_2 \dots$

$2 \cdot a\,p\,s$

$a_1 p_1 s_1 \quad a_1 p_1 s_2 \quad a_1 p_1 s_3 \quad a_1 p_2 s_1 \quad a_1 p_2 s_2$

$a_1 p_2 s_3 \dots$

$2 \cdot 3 \cdot a\,p$

$a_1 p_1 \quad a_1 p_2 \quad a_1 p_3 \dots$

$2 \cdot 3 \cdot 5 \cdot a = 2 \cdot 3 \cdot 5 \cdot 3 = 90$

Anagrammi

$\boxed{\text{COMPUTER}}$ CROMUTPE

$\underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 8!$ "8 fattoriale"

CASA $\quad 4!/2!$

$\underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \quad CA_1SA_2 \quad CA_2SA_1$

$MO_1NO_2CO_3LO_4 \quad 8!/4!$

AABBBBB $\Rightarrow \dfrac{8!}{3!\cdot 5!} = \binom{8}{3}$

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$ "n su k" coefficienti binomiali

$$(a+b)^n = a^n + \dots a^{n-1}b + \dots a^{n-2}b^2 + \dots \quad + b^n$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
...

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\underbrace{(a+b)(a+b)\cdot \dots \cdot (a+b)}_{n \text{ volte}} \qquad a^k b^{n-k}$$

$$\underbrace{aaa\dots a}_{k \text{ volte}}\underbrace{bbb\dots b}_{n-k \text{ volte}} \qquad \binom{n}{k}$$

$\binom{0}{0}$

$\binom{1}{0}$ $\binom{1}{1}$

$\binom{2}{0}$ $\binom{2}{1}$ $\binom{2}{2}$

...

$$\boxed{0! = 1}$$

$$\binom{0}{0} = \frac{0!}{0!\cdot 0!} = 1$$

$$\boxed{\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}}$$

$$\boxed{\binom{n}{k} = \binom{n}{n-k}}$$

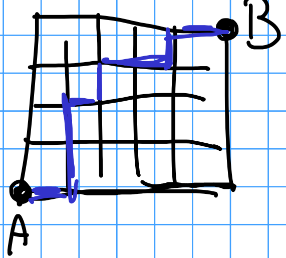
$$\binom{n}{n-k} = \frac{n!}{(n-k)!\,(\cancel{n}-(\cancel{n}-k))!} = \binom{n}{k}$$

$$\binom{n}{k} + \binom{n}{k+1} = \frac{n!}{k!\,(n-k)!} + \frac{n!}{(k+1)!\,(n-k-1)!} =$$

$$n!\left(\frac{1}{k!\,(n-k)\,(n-k-1)!} + \frac{1}{(k+1)\,k!\,(n-k-1)!}\right) =$$

$$\frac{m!}{k!(m-k-1)!}\left(\frac{1}{m-k}+\frac{1}{k+1}\right)=\frac{m!}{k!(m-k-1)!}\cdot\frac{\cancel{k}+1+m-\cancel{k}}{(m-k)(k+1)}=$$

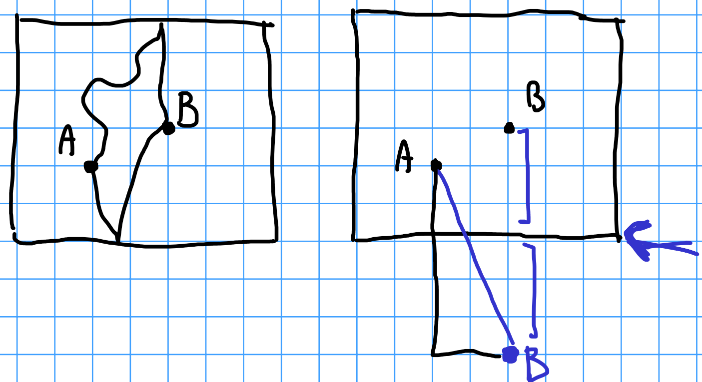
$$=\frac{(m+1)\,m!}{(k+1)k!\,(m-k)(m-k-1)!}=\frac{(m+1)!}{(k+1)!\,(m-k)!}=\binom{m+1}{k+1}$$



5 destra e 4 alto

$\binom{9}{4}$ DDDDDAAAA

DAADADDAD



$\sqrt{25+4}=\boxed{\sqrt{29}}$

Probabilitá

Dado a 6 facce

$$\frac{\text{Casi favorevoli}}{\text{Casi possibili}}=\text{Probabilitá}$$

$\frac{1}{6}$ | 2 dadi. Somma 5

1	4
2	3
3	2
4	1

$\cancel{\frac{1}{11}}$

$$\frac{4}{36}=\frac{1}{9}$$

Somma 7 $\quad \frac{1}{6} = \frac{6}{36}$

1	6	4	3
2	5	5	2
3	4	6	1

Cesto con 90 palline numerate da 10 a 99

A e B. B vince se il numero di B ha sia la cifra delle decine sia quella delle unità maggiori o uguali a quelli di A

32 $\quad$ ~~27~~ $\quad$ 47

Probabilità che B vinca?

decine $\boxed{\frac{45}{81}}$ $\quad$ 9 casi =
36 B > A
36 A > B

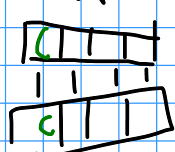
$\frac{55}{100}$ $\quad$ 10 A = B
45 A > B
45 B > A

$\frac{\cancel{45}^{5}}{\cancel{81}_{9}} \cdot \frac{\cancel{55}^{11}}{\cancel{100}_{4}} = \boxed{\frac{11}{36}}$ $\quad$ 47

16

A	B
C	C
Q	Q
F	F
P	P

A



B

C Q F P $\Rightarrow$ A
P C Q F

$\boxed{\text{C Q F P}}$ $\Rightarrow$ B
P C F Q

C Q F P
Q F P C
F P C Q
P C Q F

$\boxed{3}\,\cdot$
Q C
Q F

$\boxed{2} \cdot 3$

$4! = 24$

C Q
→ Q C $\boxed{1 \cdot 3}$

C Q
Q F $\boxed{3 \cdot 2}$ $\frac{9}{24} = \frac{3}{8}$

$1 - \frac{3}{8} = \frac{5}{8}$

C Q F P 9
P · Q · F
3 3

Esercizi

1. $m = p_1^{a_1} p_2^{a_2} \cdot \ldots \cdot p_k^{a_k}$

$(a_1+1)(a_2+1)(a_3+1) \cdot \ldots \cdot (a_k+1)$

44

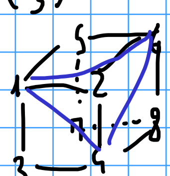
$$\begin{array}{r} 121 \\ 16 \\ \hline 726 \\ 121 \\ \hline 1936 \end{array}$$

2. $\frac{9!}{2 \cdot 2 \cdot 2}$

PORDEOÑE

$\frac{8!}{2 \cdot 2}$

3. $\binom{8}{3}$



SSSNNNNN
SNNSNSNN

$6 \cdot \binom{4}{3}$

$\binom{8}{3} - 6\binom{4}{3} = 8 \cdot 7 - 24 = 32$

$$\binom{20}{3} - 12\cdot\binom{5}{3} = 20\cdot 19\cdot 3 - 120 = 60\cdot 17 = 1020$$

4.

$\underbrace{\bullet \; \bullet \; \bullet \; \bullet \; \bullet}$

1	2	3
4	5	6
7	8	9
		~~0~~

$3 \cdot \_ \; \_ \; \_ \; \_$

$5^2 \cdot 4^2 = 5^2 \cdot 2^4$

PPDD

$$3\cdot 5^2 \cdot 2^4 \cdot \binom{4}{2} = 3^2 \cdot 5^2 \cdot 2^5$$

5.

K bambini e m caramelle

BBB ... BCCC ... CC

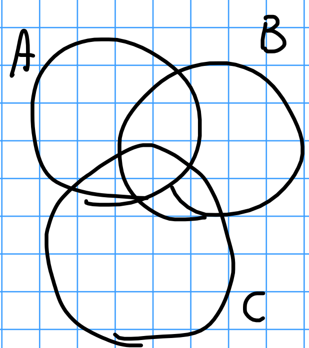
B $\underbrace{C}$ B $\uparrow$ B $\underbrace{CCC}_{}$ B $\underbrace{C}$

$\boxed{B}$ C B CC B C . . .

$$\underbrace{\binom{k-1+m}{m}}$$

$$\binom{k-1+m-k}{m-k} = \binom{m-1}{m-k}$$

6.



$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

A A A B B B C C C

$$A_1 = \{ \text{i } 3\ A \text{ sono vicini} \}$$

$$|A_1| = 9\cdot\binom{6}{3} = |A_2| = |A_3| = 9\cdot\frac{\cancel{6}\cdot5\cdot4}{\cancel{3\cdot2}} = 9\cdot20$$

$$|A_1\cup A_2\cup A_3| = |A_1|+|A_2|+|A_3| - |A_1\cap A_2| - |A_1\cap A_3| - |A_2\cap A_3| +$$
$$+ |A_1\cap A_2\cap A_3|$$

$$|A_1\cap A_2| = |A_2\cap A_3| = |A_1\cap A_3| = 9\cdot4 = 36$$

$$|A_1\cap A_2\cap A_3| = 2\cdot9 = 18$$

$$|A_1\cup A_2\cup A_3| = 9\cdot20\cdot3 - 9\cdot4\cdot3 + 9\cdot2 = 18(30-6+1) =$$
$$= 18\cdot25 = 50\cdot9 = 450$$

$$\frac{9!}{3!\,3!\,3!} = \binom{9}{3,3,3} = \frac{\cancel{9}\cdot8\cdot7\cdot6\cdot5\cdot\cancel{4}}{\cancel{3\cdot2\cdot3\cdot2}}$$

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$$\frac{\cancel{10}^{5}\cdot\cancel{50}\cdot\cancel{9}^{3}}{8\cdot7\cdot\cancel{6}_{3}\cdot\cancel{5}} = \frac{15}{56}. \qquad 1-\frac{15}{56} = \frac{41}{56}.$$