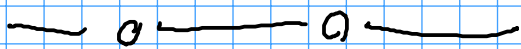


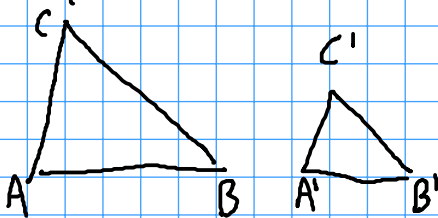
GEOMETRIA

Fabio

[Orari: 9 - 12
14.15 - 17.15]



Criteri di similitudine



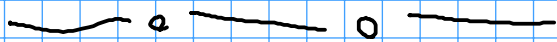
$$\begin{aligned}\hat{C}AB &= \hat{C}'A'B' \\ \hat{A}CB &= \hat{A}'C'B' \\ \hat{ABC} &= \hat{A}'B'C'\end{aligned}$$

$$AC : A'C' = AB : A'B' = BC : B'C'$$

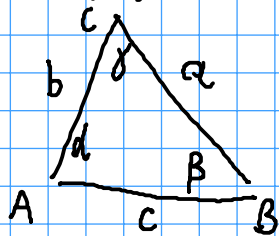
I) 2 angoli congruenti

II) $\hat{B}AC = \hat{B}'A'C'$ e $\frac{AC}{A'C'} = \frac{AB}{A'B'}$

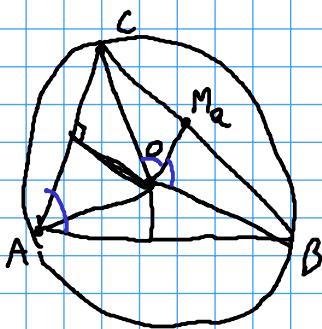
III) $\frac{AC}{A'C'} = \frac{AB}{A'B'} = \frac{BC}{B'C'}$



Punti notevoli dei triangoli



CIRCOCENTRO (assi)



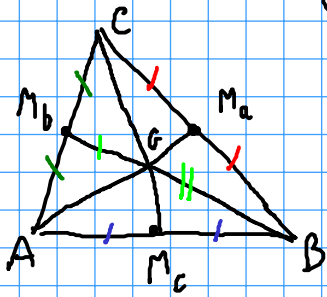
$$OA = OB = OC = R$$

$$\hat{COM}_a = \alpha$$

$$\hat{OCB} = 90 - \alpha$$

$$\hat{OCA} = 90 - \beta$$

BARICENTRO (mediane)

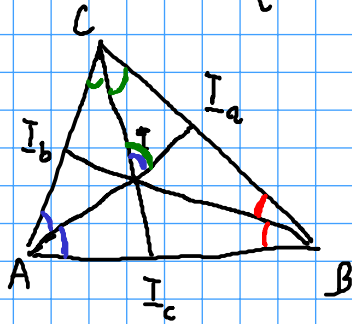


$$BG = 2GM_b$$

$$AG = 2GM_a$$

$$GC = 2GM_c$$

INCENTRO (bisettrici)

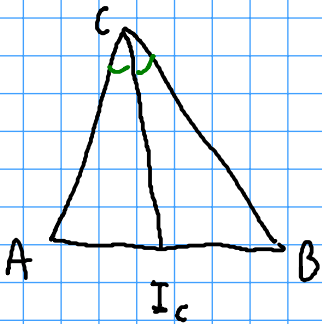


$$\begin{aligned} \text{blue} &= \alpha/2 \\ \text{red} &= \beta/2 \\ \text{green} &= \gamma/2 \end{aligned}$$

$$\begin{aligned} \hat{A}I_aC &= 180 - \gamma - \frac{\alpha}{2} \\ &= \beta + \frac{\alpha}{2} \end{aligned}$$

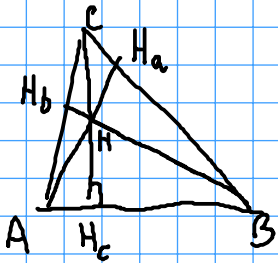
$$\begin{aligned} \hat{I_a}I_c &= 180 - \gamma - \beta - \frac{\alpha}{2} \\ &= \frac{\alpha + \gamma}{2} \end{aligned}$$

Teorema delle bisettrici



$$\frac{AI_c}{BI_c} = \frac{AC}{BC}$$

ORTOCENTRO (altezze)

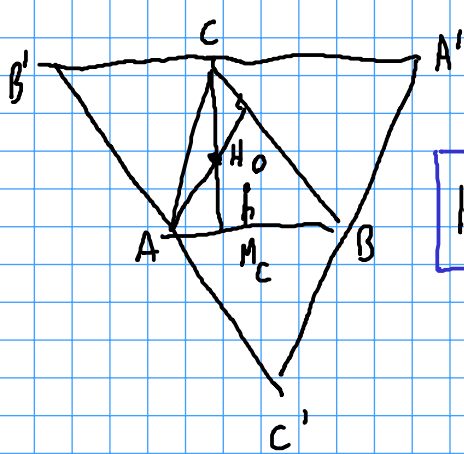


$$\hat{A}CH_c = 90 - \alpha = \hat{A}BH_b$$

$$\hat{C}HH_b = \alpha$$

$$\hat{C}HH_a = \beta$$

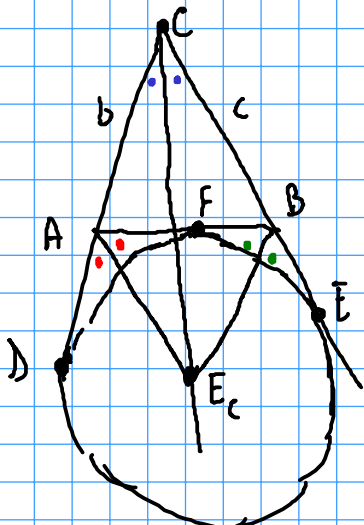
$$\hat{B}HH_a = \gamma$$



$$CH = 2OM_c$$

H è circoncentro di A'B'C'

EXCENTRI



$$CD = CE$$

$$AF = AD$$

$$BF = BE$$

$$2CD = CD + CE = b + AD + c + BE =$$

$$= b + c + AF + BF = a + b + c$$

Esercizio

$$5 \cdot 180 + 3 \cdot 180$$

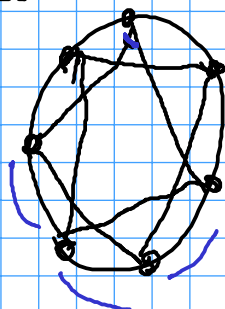
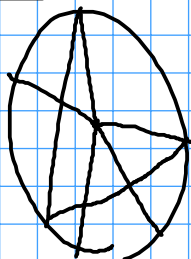
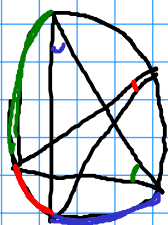
$$x = \bullet$$

$$\begin{cases} 12 + 4 \bullet + 2x = 5 \cdot 180 \\ 12 + 4 \bullet + x = 3 \cdot 180 \end{cases}$$

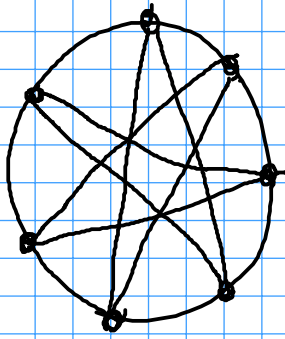
$$12 + 4 \bullet + x = 3 \cdot 180$$

$$x = 2 \cdot 180$$

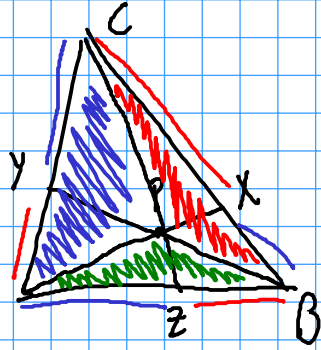
$$12 + 4 \bullet = 180$$



$$3 \cdot 180$$



TEOREMA DI CEVA



$$\frac{AZ \cdot BX \cdot CY}{BZ \cdot CX \cdot AY} = 1$$

[DEF]
" area di DEF

$$\text{Dim: } \frac{AZ}{BZ} = \frac{[ZAC]}{[ZBC]} = \frac{[PAZ]}{[PBZ]} = \frac{[ZAC] - [PAZ]}{[ZBC] - [PBZ]} = \frac{[CPA]}{[CPB]}$$

$$\frac{a}{b} = \frac{c}{d} \quad \leftarrow \quad \frac{a}{b} = \frac{a-c}{b-d}$$

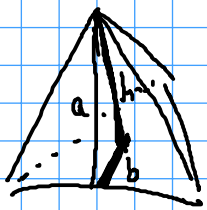
$$\frac{AZ}{BZ} = \frac{BX}{CX} = \frac{CY}{AY}$$

Esercizi: 10 biglie di raggio 10cm



Quanto è alta la piramide?

Ⓐ Tetraedro di lato 2

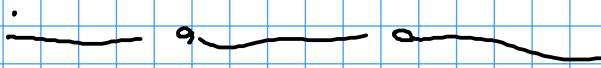
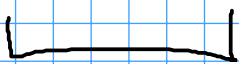


$$a = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$b = \frac{\sqrt{3}}{3}$$

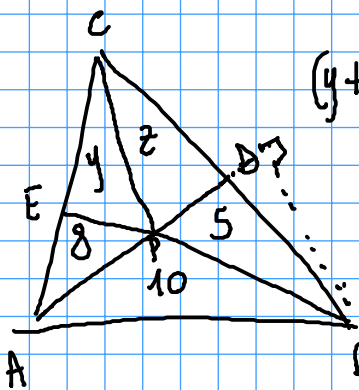
$$h = \sqrt{3 - \frac{3}{9}} = \frac{1}{3} 2\sqrt{6} = \frac{2\sqrt{6}}{3}$$

$$2 + 2 \frac{2\sqrt{6}}{3}$$



Esercizi.

1.



$$(y+8):z = AP:PD = 10:5$$

$$y+8=2z \Rightarrow y=2z-8$$

$$(z+5):y = PB:PE = 10:8 = 5/4$$

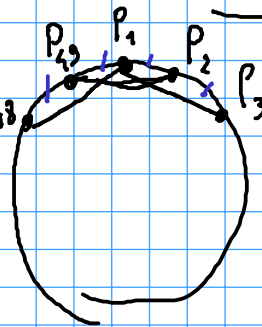
$$5y = 4z + 20$$

$$10z - 40 = 4z + 20 \Rightarrow z = 10$$

$$y = 12$$

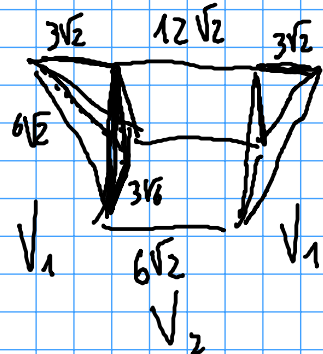
$$x = 22$$

2.



$$45 \cdot 180 = 8100$$

3.



Volume ? $V_2 + 2V_1$

$$3\sqrt{6}$$

$$3\sqrt{6}$$



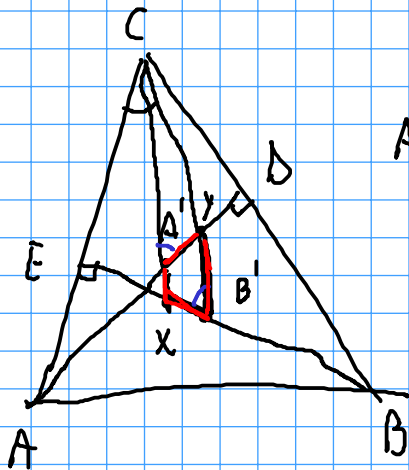
$$3\sqrt{2} \sqrt{3-1} = 6$$

$$\text{Base} = \frac{6\sqrt{2} \cdot 6}{2} = 18\sqrt{2}$$

$$V_2 = 18\sqrt{2} \cdot 6\sqrt{2} = 216 \quad V_2 + 2V_1 = 216 + 172 = 288.$$

$$V_1 = \frac{18\sqrt{2} \cdot 3\sqrt{2}}{3} = 36$$

4.



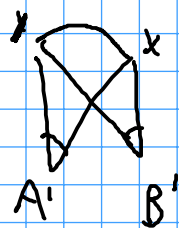
$A'DB'E$ é cíclico

$ADC \cap BEC$

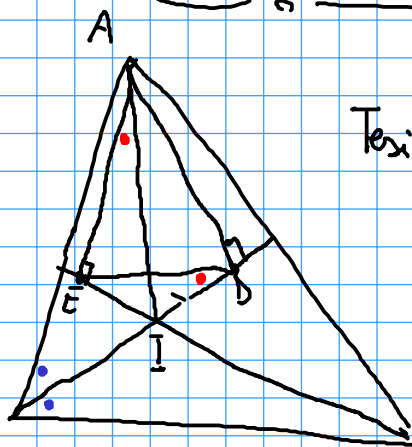
$$\frac{CD}{CE} = \frac{AD}{BE} = \frac{\sphericalangle A'D}{\sphericalangle B'E}$$

$CA'D \cap B'EC$

$$\hat{CA'D} = \hat{CB'E}$$



5.



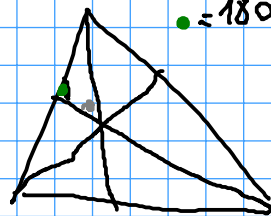
Terxi: $\hat{EDB} = \beta/2$

$AEID$ é cíclico

$$\hat{EAI} = \hat{EDB}$$

$$\hat{EAI} = 90 - \hat{EIA} = \beta/2$$

$$\hat{EIA} = 180 - \frac{\alpha}{2} - (180 - \alpha - \frac{\gamma}{2}) = \frac{\alpha + \gamma}{2}$$



$$= 180 - \alpha - \frac{\gamma}{2}$$

□