COMBINATORIA

8/2/2018

Menù: 4 antipasti: a_1 a_2 a_3 a_4
3 primi: p_1 p_2 p_3
3 secondi: s_1 s_2 s_3
2 dessert: d_1 d_2

$a_1 p_1 s_1 d_1$ $a_1 p_1 s_1 d_2$ $a_1 p_1 s_2 d_1$ $a_1 p_1 s_2 d_2$

terne $\cdot 2$

$a_1 p_1 s_1$ $a_1 p_1 s_2$ $a_1 p_1 s_3$ $a_1 p_2 s_1$

Coppie $\cdot 3 \cdot 2$

$a_1 p_1$ $a_1 p_2$ $a_1 p_3$ $a_2 p_1$ $a_2 p_2$ ...

antipasti $\cdot 3 \cdot 3 \cdot 2 = 4 \cdot 3 \cdot 3 \cdot 2 = 72$

Numero di divisori di un intero

24: 1 2 3 4 6 8 12 24

$24 = 2^3 \cdot 3^1$ $\quad 2^a \cdot 3^b$

$a = 0, 1, 2, 3 \leftarrow 4$ $\quad 8$

$b = 0, 1 \leftarrow 2$

$n = p_1^{a_1} \cdot p_2^{a_2} \cdot \ldots \cdot p_m^{a_m}$ $\quad (a_1+1)(a_2+1)\ldots(a_m+1)$

Anagrammi

SCUOLA

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

Fattoriale

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$n! = n(n-1)(n-2)\ldots \cdot 2 \cdot 1$

CA_1SA_2

CA_1SA_2 CA_2SA_1 $\frac{4!}{2!} = \frac{4 \cdot 3 \cdot 2}{2} = 12$

MONOCOLO $\frac{8!}{4!}$

MAMMA. $\frac{5!}{3! \cdot 2!} = \binom{5}{2}$ "5 su 2" coefficiente binomiale

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{n-k} = \frac{n!}{(n-k)!\,(n-(n-k))!} = \frac{n!}{(n-k)!\,k!}$$

$(n-(n-k)) = k$

$$(a+b)^n$$

$$(a+b)^5 = \binom{5}{0}a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$\binom{5}{0} = 1$ $\binom{5}{1} = 5$ $\binom{5}{2} = 10$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
→ 1 5 10 10 5 1

$(a+b)\cdot(a+b)\cdot(a+b)(a+b)(a+b)$

$1a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + \dots$

aaaab
abaaa
aaabb

$0! = 1$ $\quad \binom{5}{0} = \frac{5!}{0! \cdot 5!} = \frac{1}{0!} = 1$ $\quad \binom{0}{0} = 1 \quad \frac{0!}{0! \cdot 0!} = 1$

$10 = 6 + 4$

$\binom{5}{3} = \binom{4}{2} + \binom{4}{3}$

$\binom{m+1}{k+1} = \binom{m}{k} + \binom{m}{k+1}$

$$\binom{m}{k} + \binom{m}{k+1} = \frac{m!}{k!(m-k)!} + \frac{m!}{(k+1)!(m-k-1)!} = m! \left(\frac{1}{k!(m-k-1)!(m-k)} + \right.$$

$$\left. + \frac{1}{(k+1)\,k!(m-k-1)!} \right) = \frac{m!}{k!(m-k-1)!} \left(\frac{1}{m-k} + \frac{1}{k+1} \right) =$$

$$= \frac{m!}{k!(m-k-1)!} \cdot \frac{k+1+m-k}{(m-k)(k+1)} = \frac{(m+1)!}{(k+1)!(m-k)!} = \binom{m+1}{k+1}$$

$(m+1-(k+1))!$

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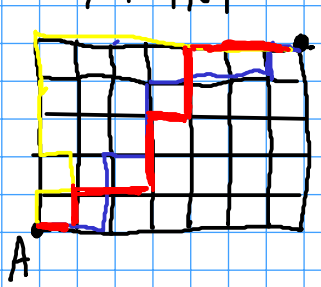
CAMMINI



DDDDDDDSSSS

DSDDSSDSSDDD

$\binom{12}{7}$

$$\binom{3}{1}\binom{8}{1}+\binom{7}{2}\binom{4}{1}$$

$$3\cdot 8 + 21\cdot 4 = 24+84 = 108$$

Sottoinsiemi

$S=\{1,2,\dots,n\}$

sottoinsiemi $= 2^n$ $\quad \{1,2,3,4,5\}$

S S ... S $\quad$ S N S S N

N N N $\quad \{1,3,4\}$

sottoinsiemi con K elementi $= ? = \binom{n}{k}$

$\underbrace{S\,S\,S\dots S}_{k \text{ volte}}\ \underbrace{N\,N\dots N}_{n-k \text{ volte}}$

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\dots+\binom{n}{n}=2^n \quad \Leftarrow$$

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k=\binom{n}{0}a^n b^0+\binom{n}{1}a^{n-1}b^1+\dots$$

$$+\dots+\binom{n}{n}a^0 b^n$$

$a=b=1$

$$(1+1)^n=\sum_{k=0}^{n}\binom{n}{k}\cdot 1=\binom{n}{0}+\binom{n}{1}+\dots+\binom{n}{n}$$

Bambini e caramelle

5 $\qquad$ 20

BBBBBCCC...CC

CBCCBCCCCCBCBBCCC...C

BBBBCCC...C
20

BB

$\binom{4+20}{20} = \binom{24}{20} = \frac{24 \cdot 23 \cdot 22 \cdot 21 \cdot 20!}{4! \, 20!} =$

$= 23 \cdot 22 \cdot 21$

Esercizi

1. 2^6 . $\{1, 2, \dots, 24\}$

$\{1, 2, \dots, 13\}$ $\frac{13 \cdot 14}{2} = 13 \cdot 7$ 2^{12}

$\{2, \dots, 24\}$ 2^{23}

A C

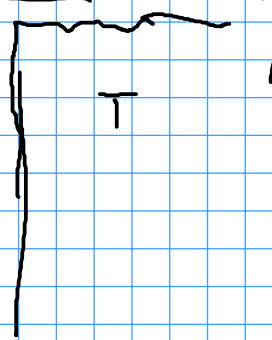
A $A \cup \{1\}$ C $C \cup \{1\}$

dispari pari

X X

2.

$81!$



$64 \cdot 49 \cdot 36 \cdot 25 \cdot 16 \cdot 9 \cdot 4 \cdot 1$

$= \frac{(8!)^2}{8!} = 8!$

3. $0 < x_1 < x_2 < x_3 < x_4 < x_5 < 2017$

$\binom{2016}{5}$

— o — o —

4. 2^6

$1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$1^{\text{é}}$ 2^{e} $\dots$ 6 7^{e}

V S V V S V

V S 2^6

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5. $\sum_{k=0}^{m} \binom{m}{k}^2 = \sum_{k=0}^{m} \binom{m}{k}\binom{m}{m-k} = \binom{2m}{m}$

$\left(\sum_{k=0}^{m} \binom{m}{k} = 2^m\right)$

k	$m-k$
0	m
1	$m-1$
$\vdots$	
m	0

$\binom{m}{0}\binom{m}{1} \cdots \binom{m}{k} \cdots \binom{m}{m}$ — $\binom{2m}{m}$

$\sum_{i=0}^{k} \binom{m}{i}\binom{m}{k-i} = \binom{m+m}{k}$

m | m

i $\quad k-i$

— o — c —

$2m$

k $\quad m-k$

$\binom{m}{k} \cdot \binom{m}{m-k}$ $\quad \binom{2m}{m}$ $\quad m$ | m

( )

— o — c —